Lecture Two

October 10,2002

- Waves
- Waves Properties
- Wave Motion
- Oscillation
- Electrical Circuit Analogy

What is a wave ?

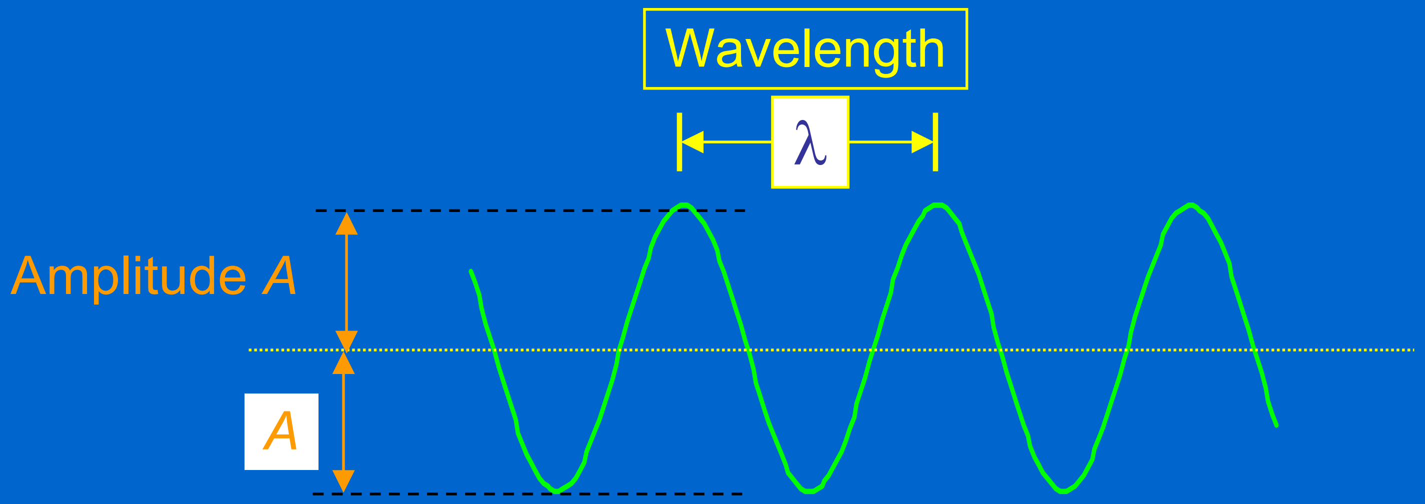
- One definition:
  - A wave is a traveling disturbance that transports energy but not matter.
- Examples:
  - Sound waves (air moves back & forth)
  - Stadium waves (people move up & down)
  - Water waves (water moves up & down)
  - Light waves (what moves ??)

Types of Waves

- **Transverse:** The medium oscillates perpendicular to the direction the wave is moving.
  - Water (more or less)
  - Guitar String
- **Longitudinal:** The medium oscillates in the same direction as the wave is moving
  - Sound
  - Slinky

Wave Properties

- Wavelength: The distance between identical points on the wave.
- Amplitude: The maximum displacement *A* of a point on the wave.



Wave Properties...

- Period: The time T for a point on the wave to undergo one complete oscillation.

- Speed: The wave moves one wavelength λ in one period T so its speed is $v = \lambda / T$.

$$\boxed{v = \frac{\lambda}{T}}$$

$v = \lambda / T$

Wave Properties...

- We will show that the speed of a wave is a *constant* that depends only on the medium, not on amplitude, wavelength or period

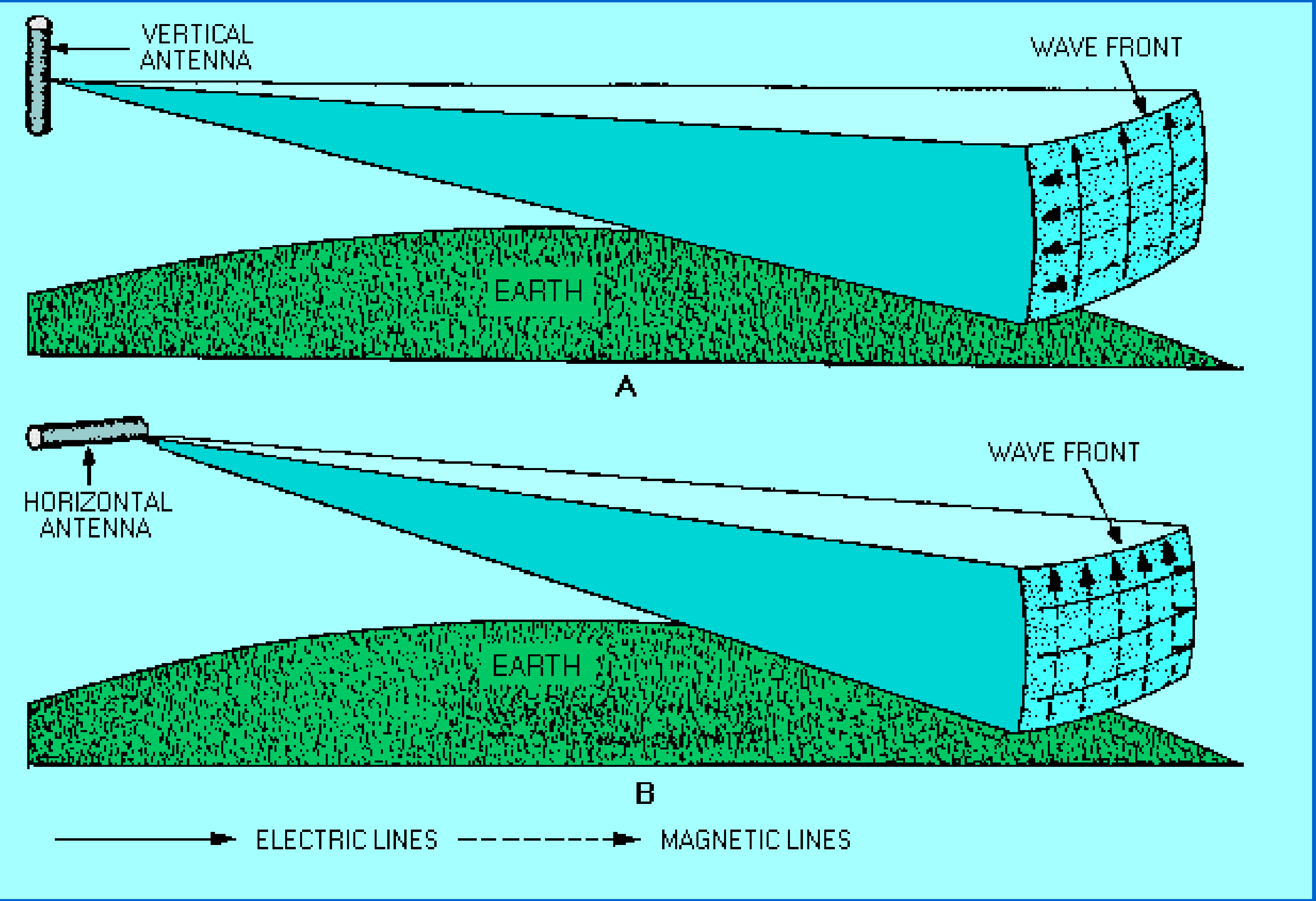
λ and T are related !

- $\lambda = v\,T$ or $\lambda = 2\pi v / \omega$ (since $T = 2\pi / \omega$)

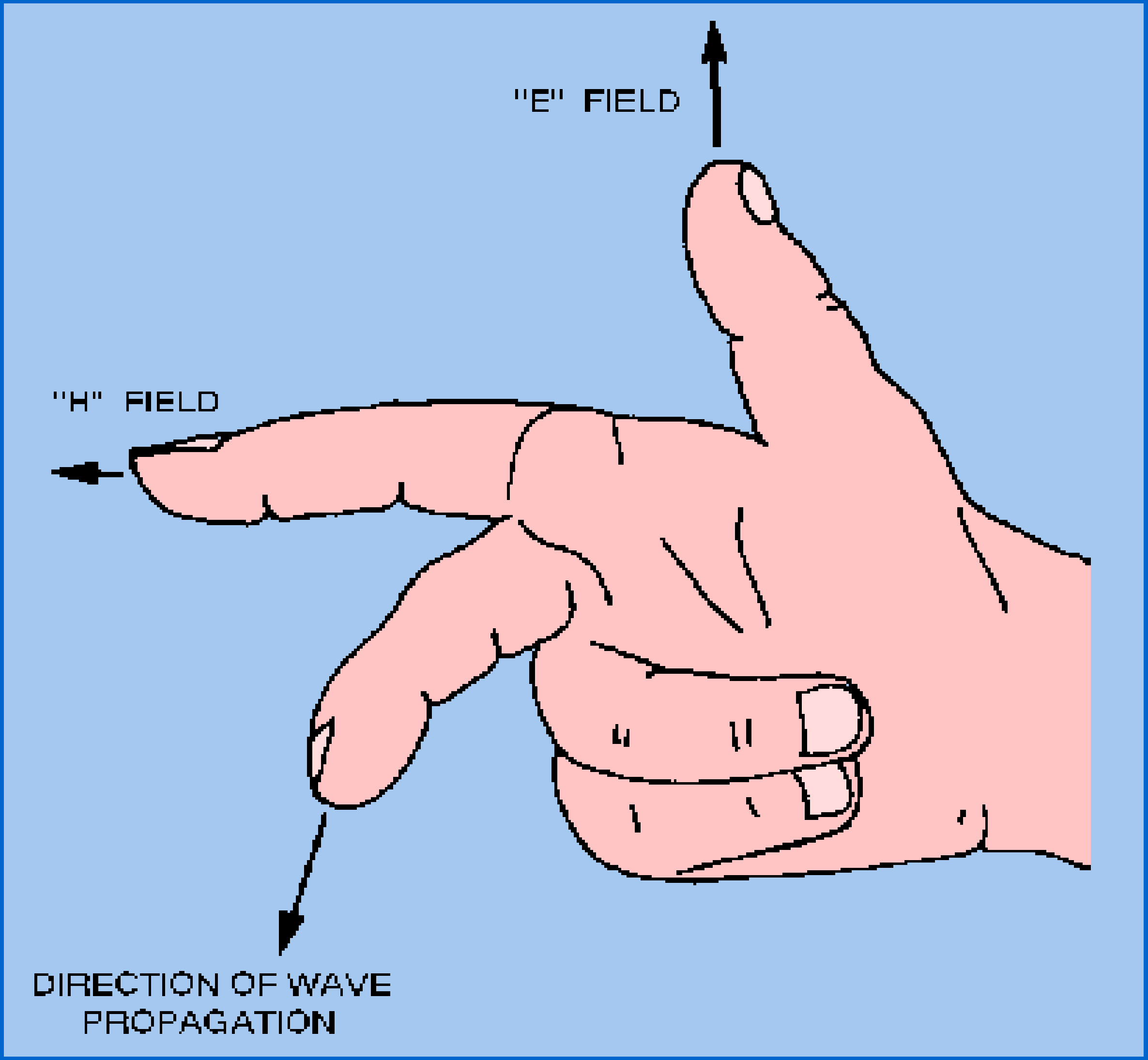
  or $\lambda = v / f$ (since $T = 1/ f$)

- Recall *f = cycles/sec* or *revolutions/sec*

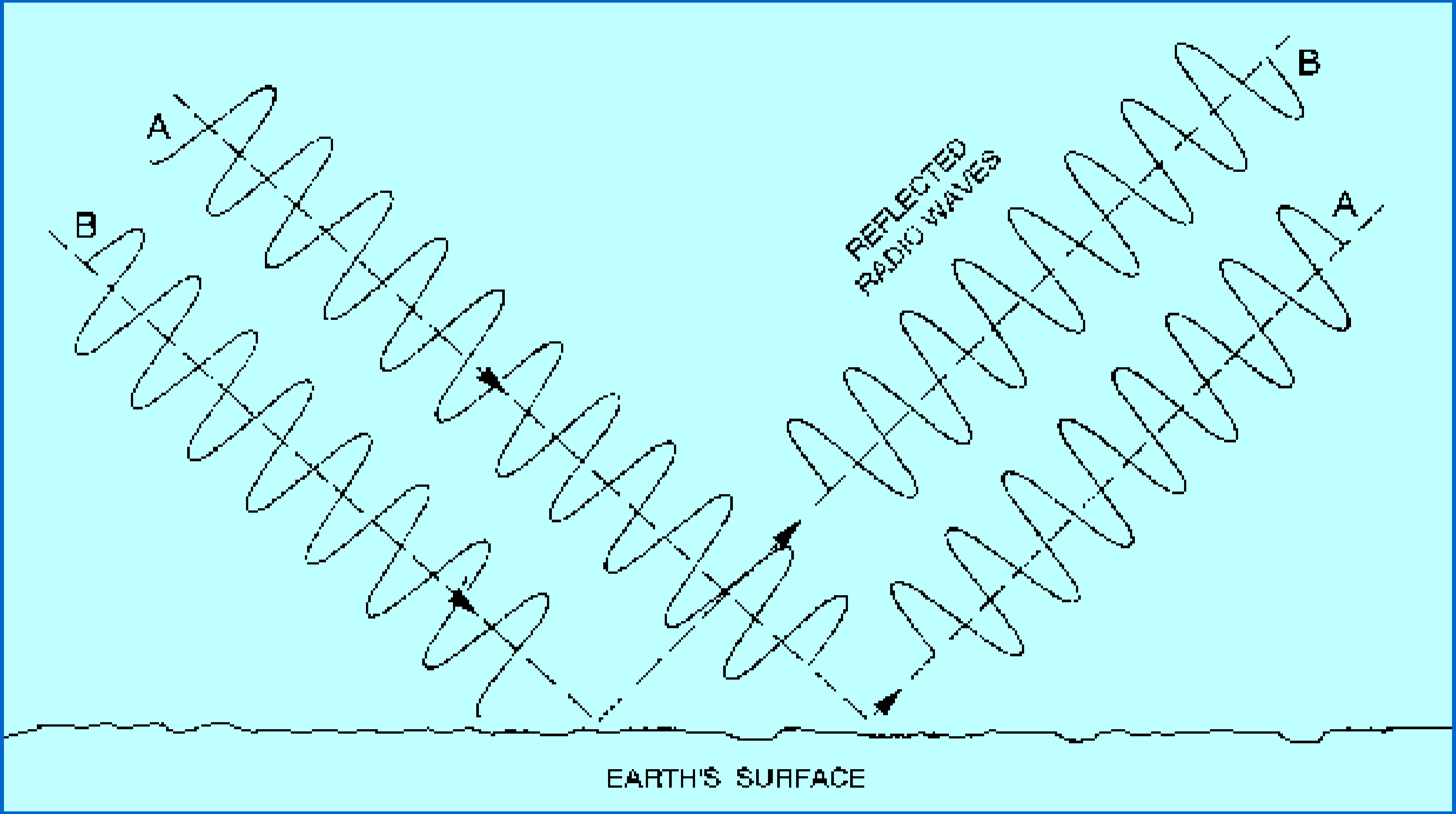
$\omega = rad/sec = 2\pi f$


VERTICAL ANTENNA
WAVE FRONT
EARTH
A
HORIZONTAL ANTENNA
WAVE FRONT
EARTH
B
ELECTRIC LINES
MAGNETIC LINES

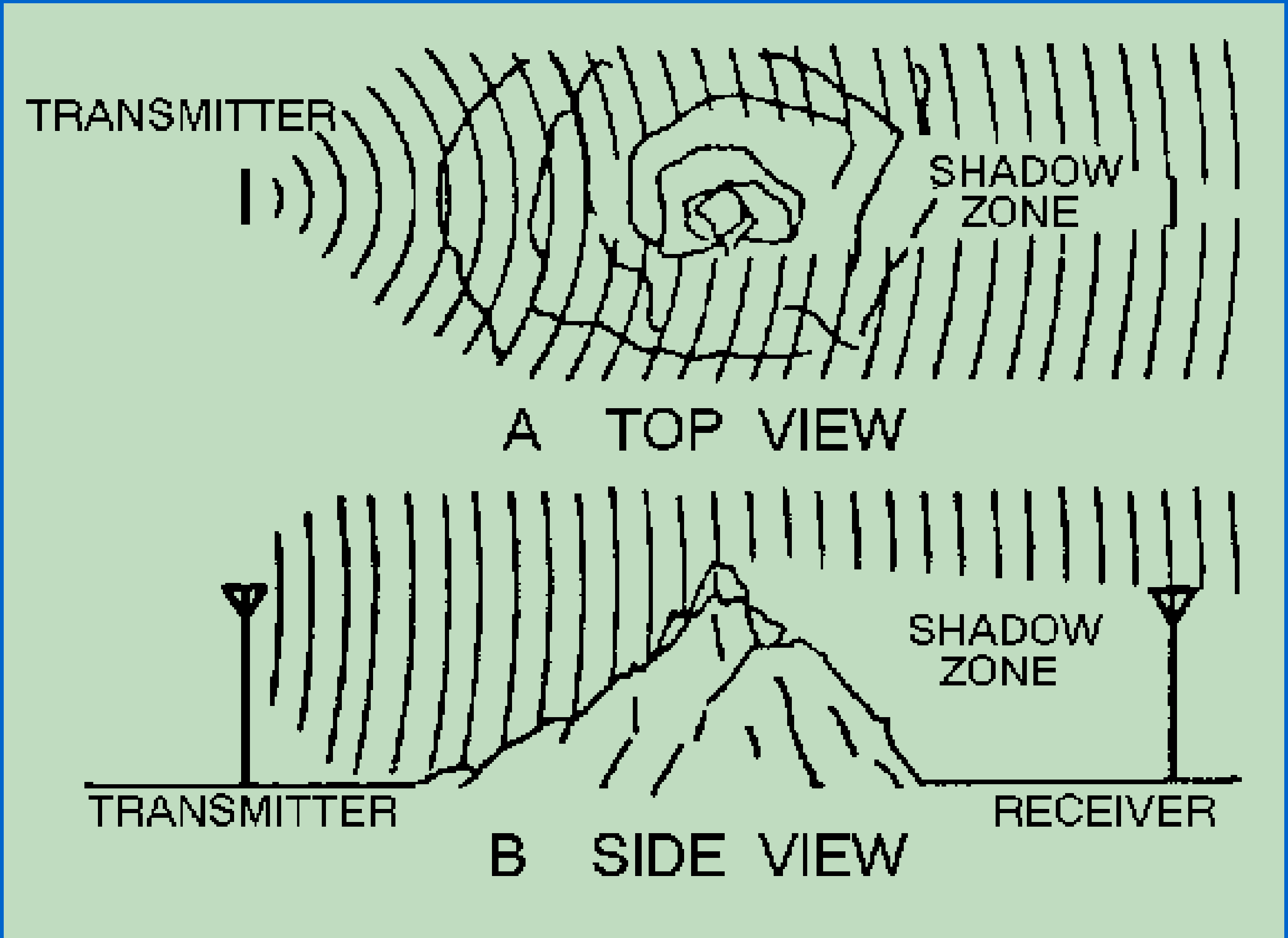
Right-hand rule for propagation

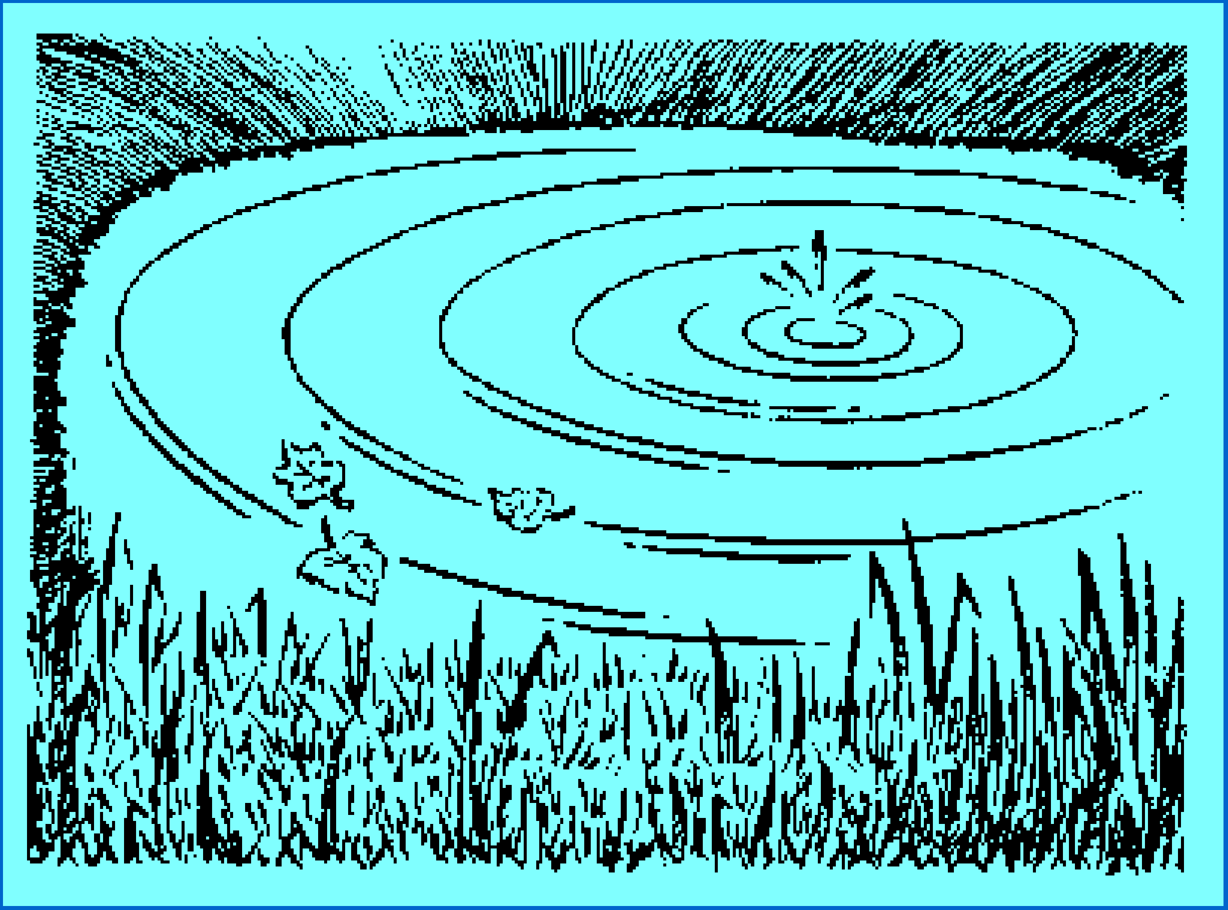


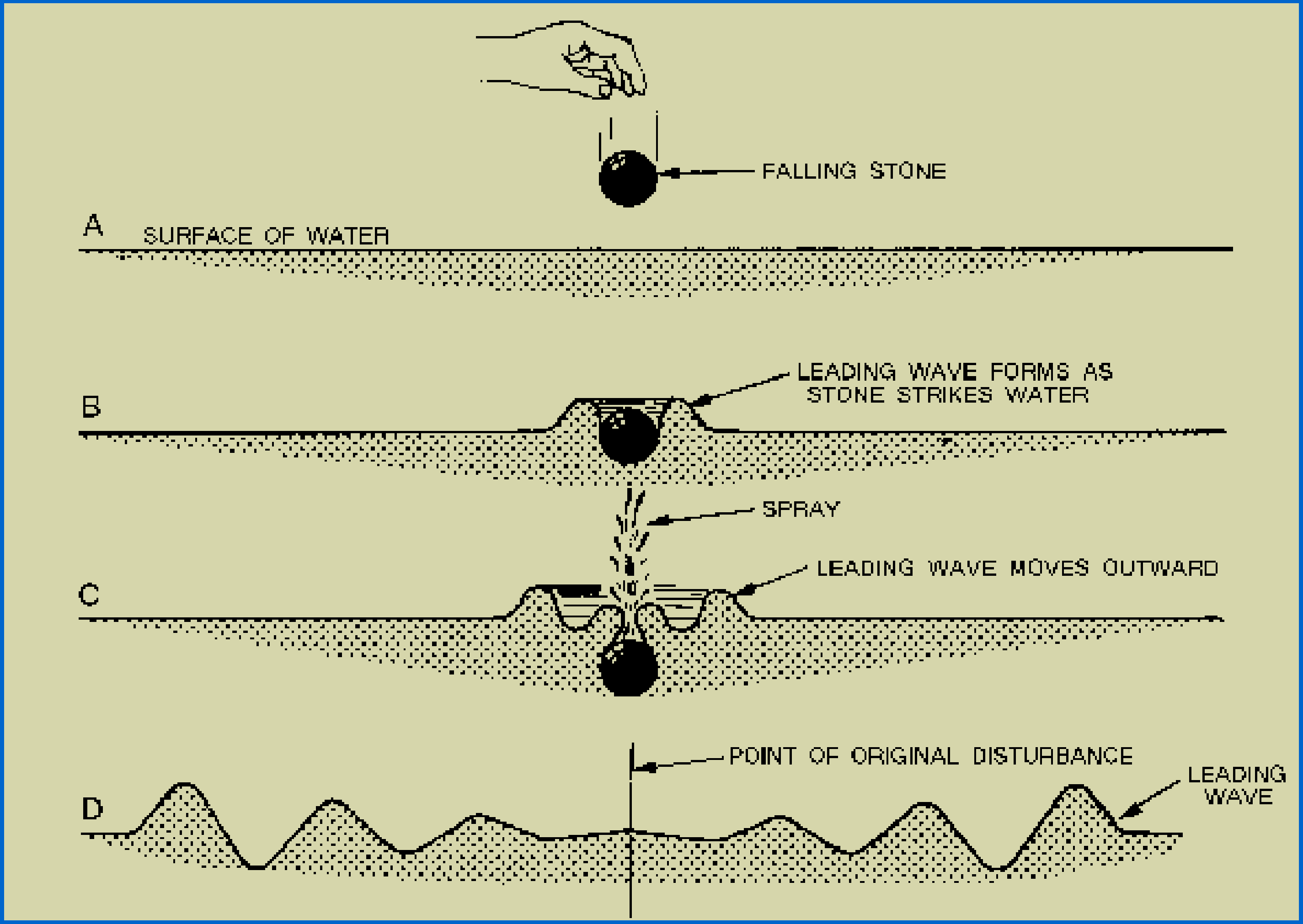
Phase shift of reflected radio waves

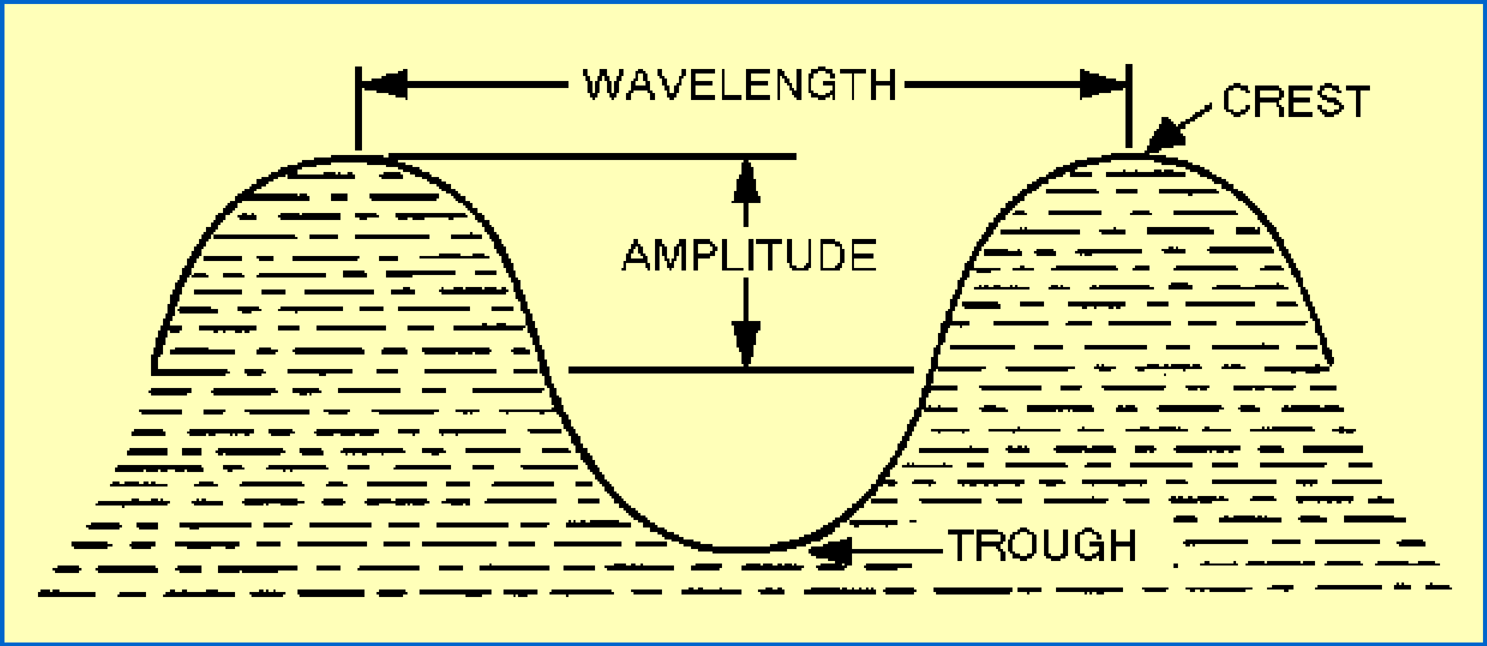


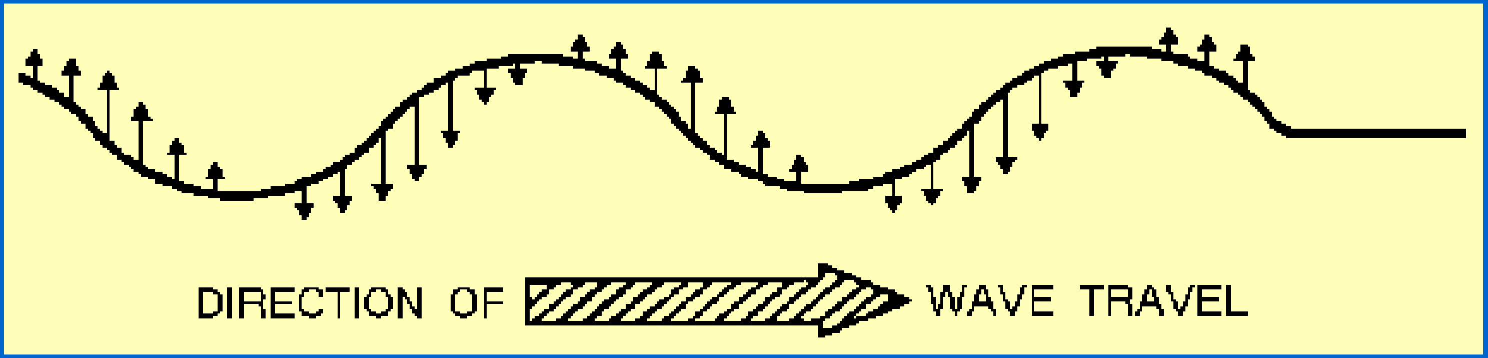
Diffraction around an object.

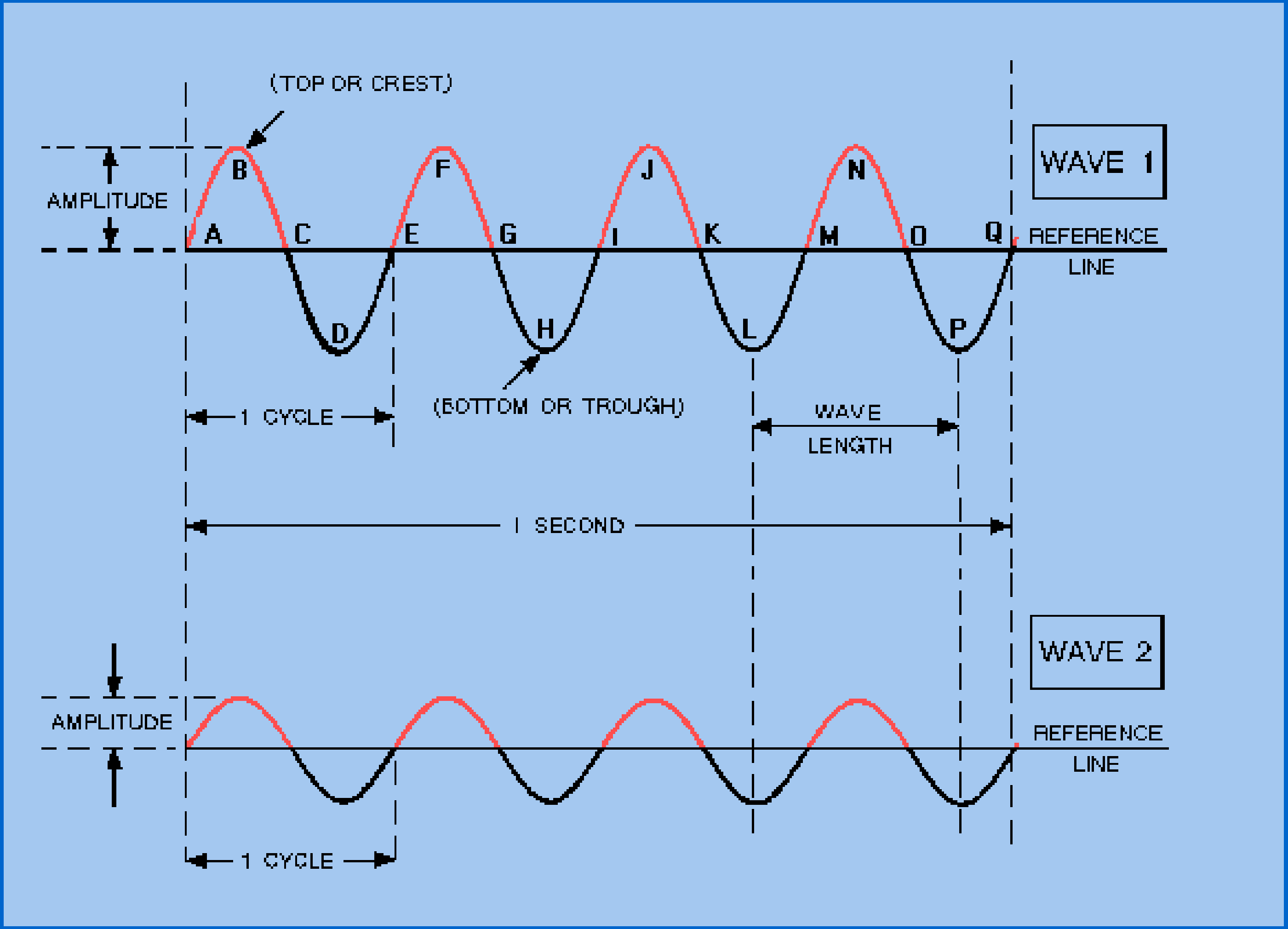




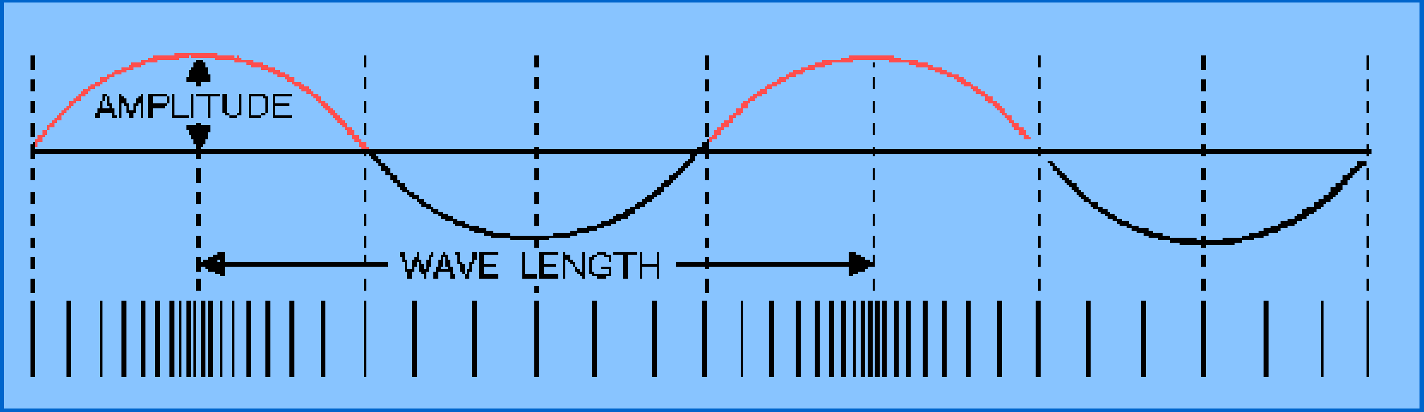

FALLING STONE
A
SURFACE OF WATER
LEADING WAVE FORMS AS
STONE STRIKES WATER
B
SPRAY
LEADING WAVE MOVES OUTWARD
C
POINT OF ORIGINAL DISTURBANCE
LEADING
WAVE
D


WAVELENGTH
CREST
AMPLITUDE
TROUGH


DIRECTION OF
WAVE TRAVEL


(TOP OR CREST)
AMPLITUDE
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
WAVE 1
REFERENCE LINE
(BOTTOM OR TROUGH)
1 CYCLE
WAVE LENGTH
1 SECOND
WAVE 2
AMPLITUDE
REFERENCE LINE
1 CYCLE

Longitudinal wave represented graphically by a transverse wave



Wave Motion

- The speed of sound in air is a bit over *300 m/s*, and the speed of light in air is about *300,000,000 m/s*.
- Suppose we make a sound wave and a light wave that both have a wavelength of *3 meters*.
  - What is the ratio of the frequency of the light wave to that of the sound wave ?

(a) About 1,000,000

(b) About .000,001

(c) About 1000

Solution

- We have shown that $v = \lambda / T = \lambda f$ (since $f = 1 / T$)

So $f = \dfrac{v}{\lambda}$

Since λ is the same in both cases, and $\dfrac{v_{light}}{v_{sound}} \cong 1{,}000{,}000$

$$\boxed{\frac{f_{light}}{f_{sound}} \cong 1{,}000{,}000}$$

Solution

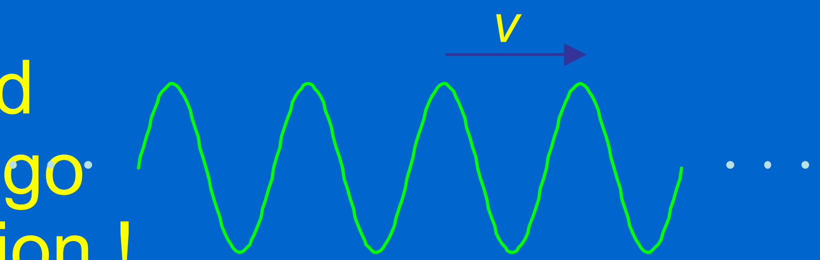
- What are these frequencies ???

For sound having λ = *3m* : $f = \frac{v}{\lambda} \approx \frac{300\, m/s}{3m} = 100\ Hz$ (low humm)

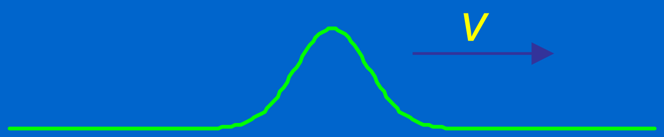
For light having λ = *3m* : $f = \frac{v}{\lambda} \approx \frac{3 \times 10^{8}\, m/s}{3m} = 100\ MHz$ (FM radio)

Wave Forms

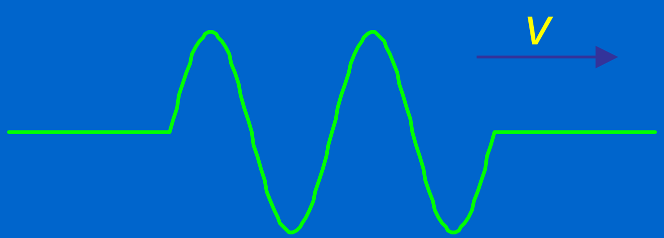
- So far we have examined "continuous waves" that go on forever in each direction !



  - We can also have "pulses" caused by a brief disturbance of the medium:



  - And "pulse trains" which are somewhere in between.



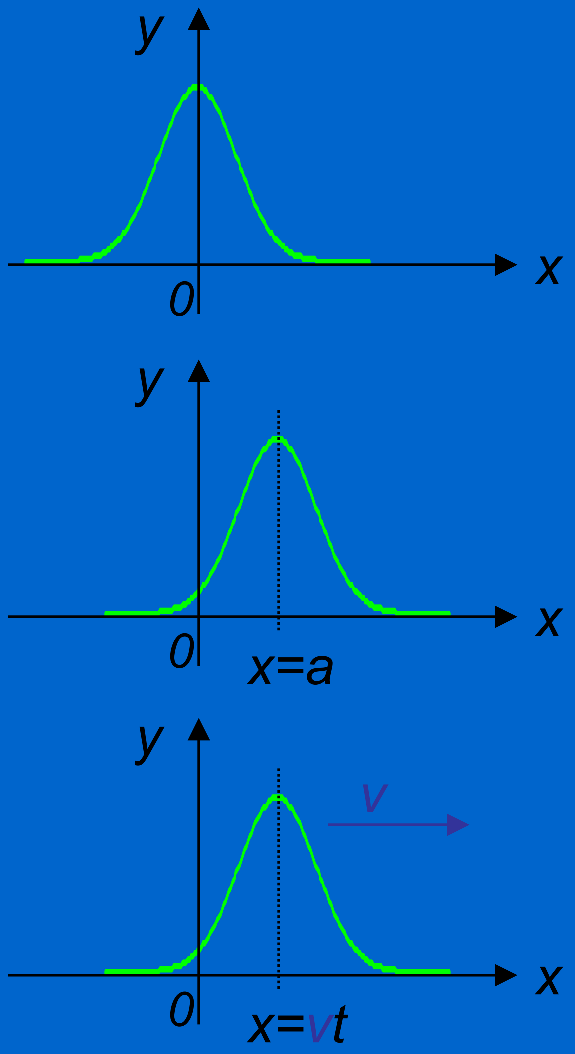
Mathematical Description

- Suppose we have some function $y = f(x)$:

- $f(x-a)$ is just the same shape moved a distance a to the right:

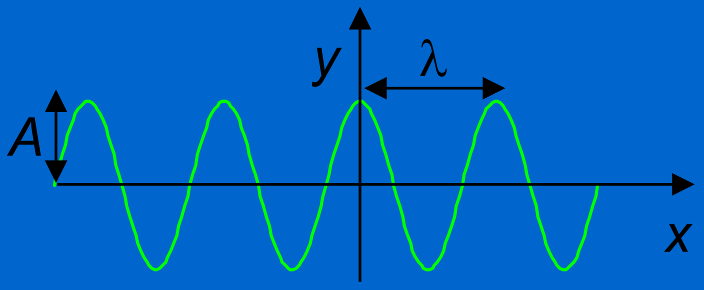
- Let $a=vt$ Then

  $f(x-vt)$ will describe the same shape moving to the right with speed v.



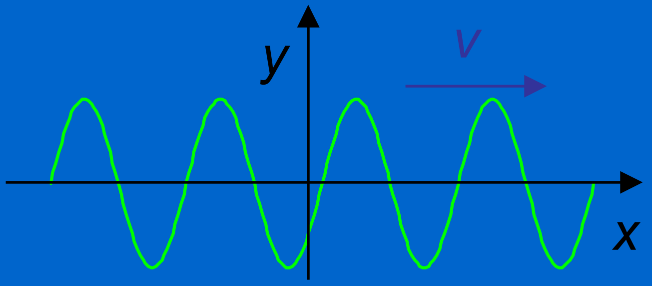
Math...

- Consider a wave that is harmonic in *x* and has a wavelength of λ.



If the amplitude is maximum at *x=0* this has the functional form:

$$y(x) = A\cos\left(\frac{2\pi}{\lambda}x\right)$$



- Now, if this is moving to the right with speed *v* it will be described by:

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

Math...

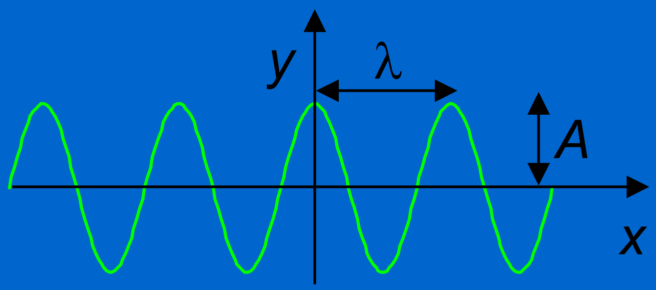
- So we see that a simple harmonic wave moving with speed *v* in the *x* direction is described by the equation: $$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

- By using $v = \frac{\lambda}{T} = \frac{\lambda\omega}{2\pi}$ from before, and by defining $\boxed{k \equiv \frac{2\pi}{\lambda}}$

  we can write this as: $$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

Math Summary

- The formula $y(x,t) = A\cos(kx - \omega t)$ describes a harmonic wave of amplitude *A* moving in the +*x* direction.



- Each point on the wave oscillates in the *y* direction with simple harmonic motion of angular frequency ω.
- The wavelength of the wave is $\lambda = \dfrac{2\pi}{k}$
- The speed of the wave is $v = \dfrac{\omega}{k}$
- The quantity *k* is often called “wave number”.

Wave Motion

- A harmonic wave moving in the positive x direction can be described by the equation $y(x,t) = A\cos(kx - \omega t)$
- Which of the following equation describes a harmonic wave moving in the negative x direction ?

(a) $y(x,t) = A\sin(kx - \omega t)$

(b) $y(x,t) = A\cos(kx + \omega t)$

(c) $y(x,t) = A\cos(-kx + \omega t)$

Solution

- Recall $y(x,t) = A\cos(kx - \omega t)$ came from

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

- The sign of the term containing the *t* determines the direction of propagation.
- We change the sign to change the direction:

$y(x,t) = A\cos(kx - \omega t)$ moving toward + x

$\downarrow$

$y(x,t) = A\cos(kx + \omega t)$ moving toward - x

Solution

- Recall $y(x,t) = A\cos(kx - \omega t)$ came from

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

- Actually , it's the <u>relative</u> sign between the term containing the *x* and the term containing the *v*:

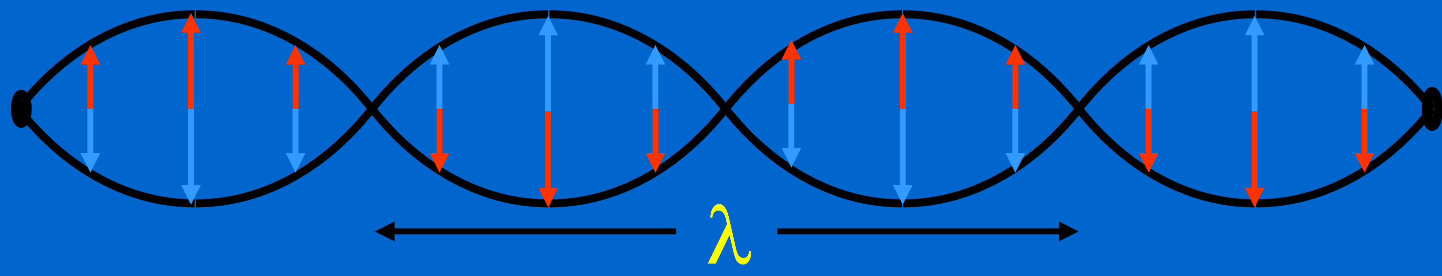
$y(x,t) = A\cos(kx - \omega t)$ moving toward + x

$y(x,t) = A\cos(-kx + \omega t) = A\cos(-(kx - \omega t))$

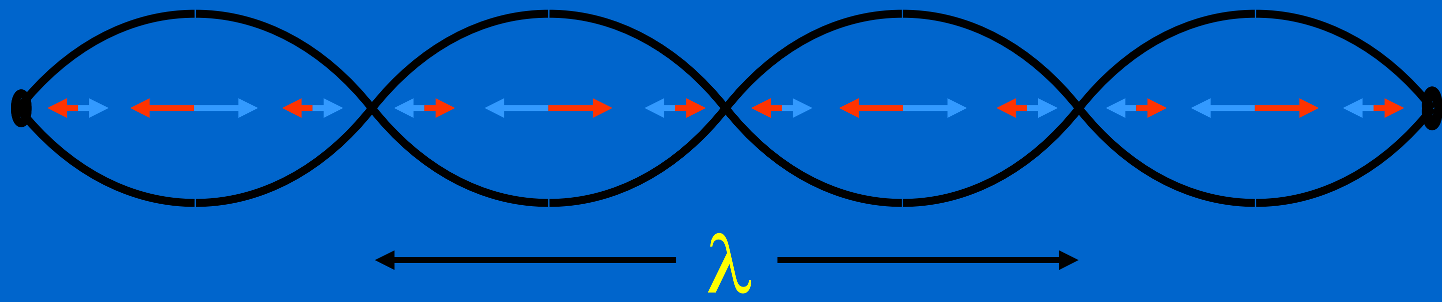
$= A\cos(kx - \omega t)$ also moving toward + x

Standing Waves:

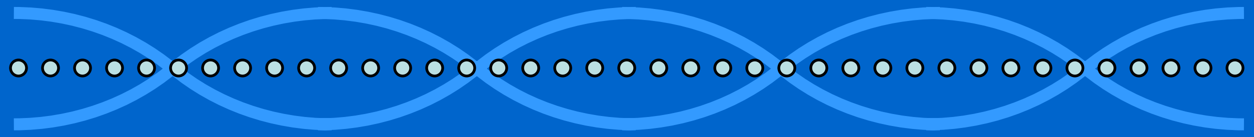
Transverse: $v = \lambda f$



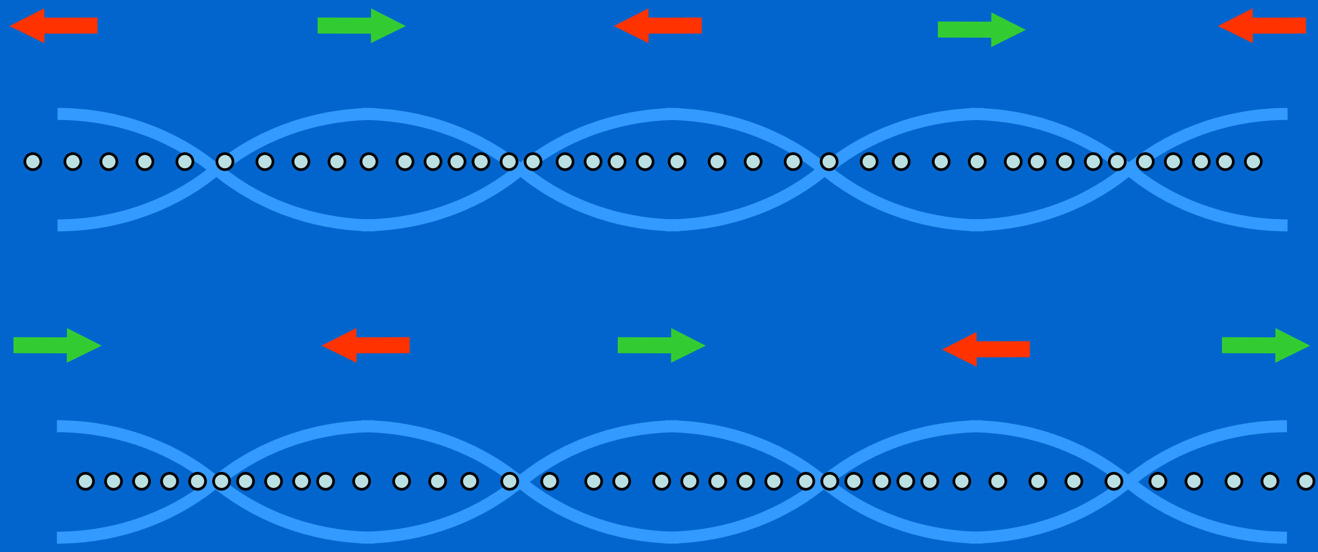
Longitudinal: $v = \lambda f$



Longitudinal standing waves

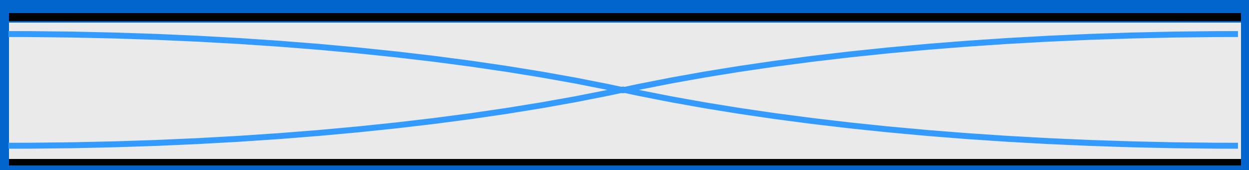


Longitudinal standing waves (ex. antinodes at each end)



Pipes: 2 Types

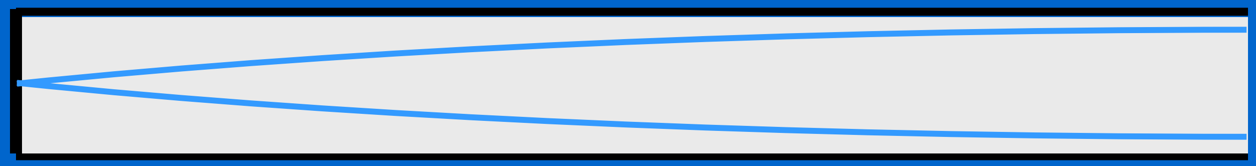
Open Pipe (open on both ends)



$\lambda_1 = 2L$

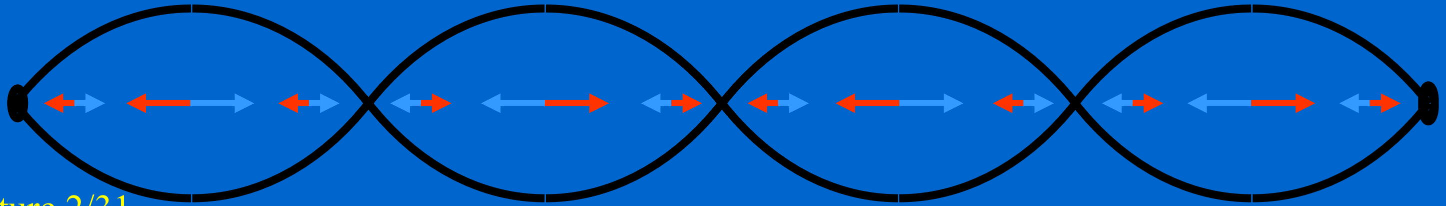
$f_1 = v/2L$

Closed Pipe (one open, one closed end)



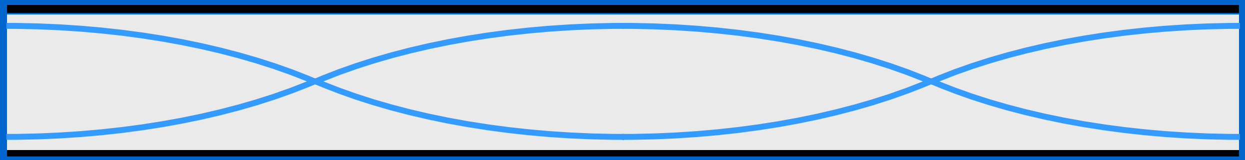
$\lambda_1 = 4L$

$f_1 = v/4L$



Next highest frequencies

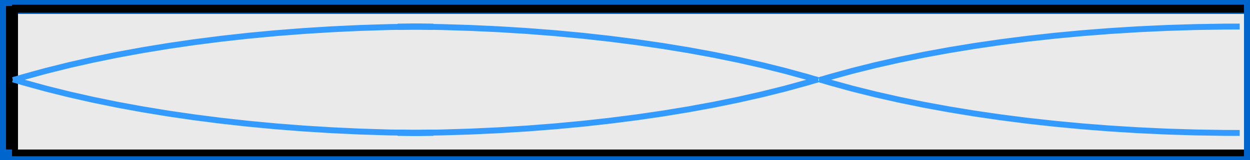
Open Pipe (open on both ends)



$\lambda_2 = L$

$f_2 = v/L = 2f_1$

Closed Pipe (one open, one closed end)

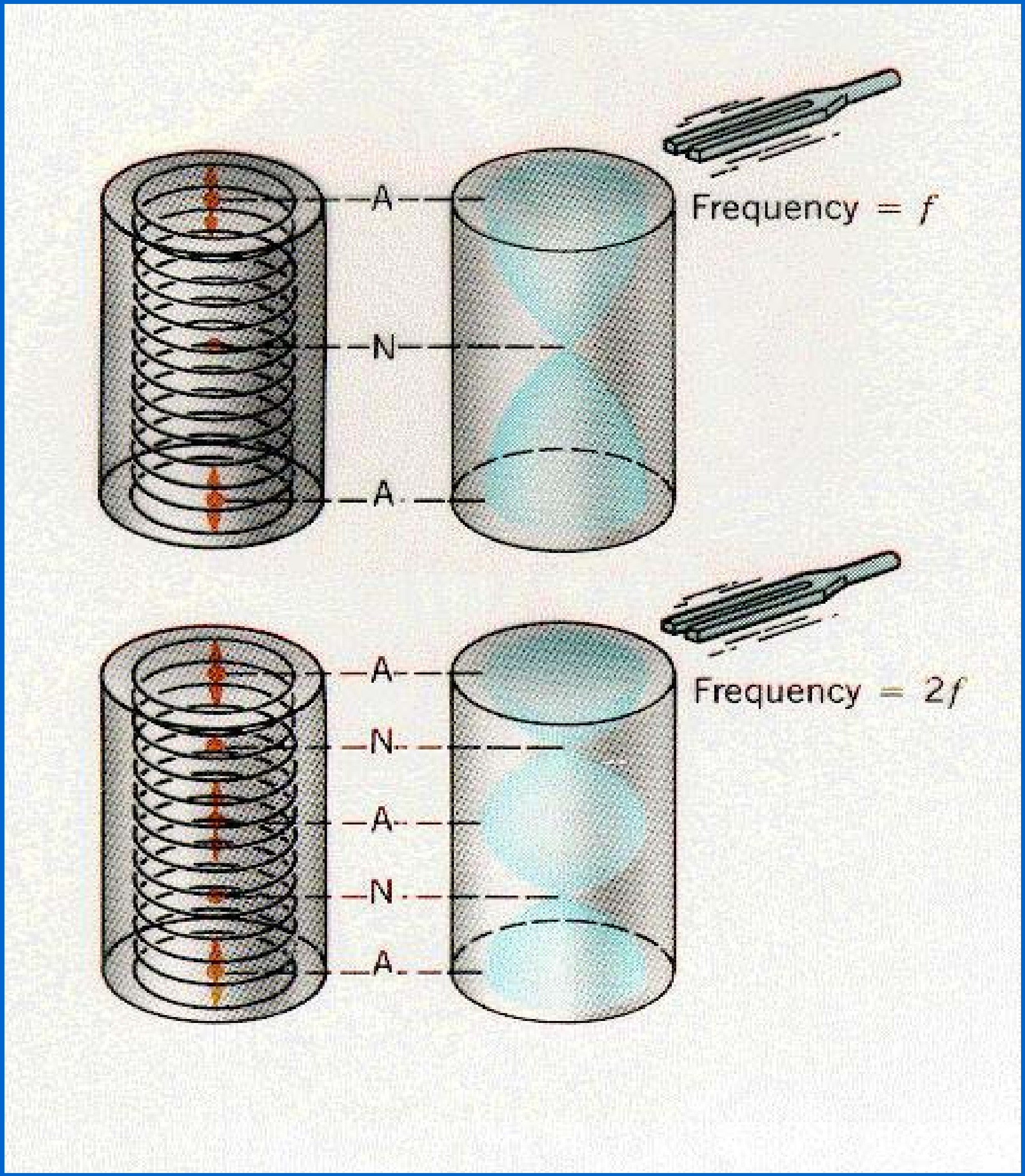


$\lambda_3 = (4/3)L$

$f_3 = 3v/4L = 3f_1$

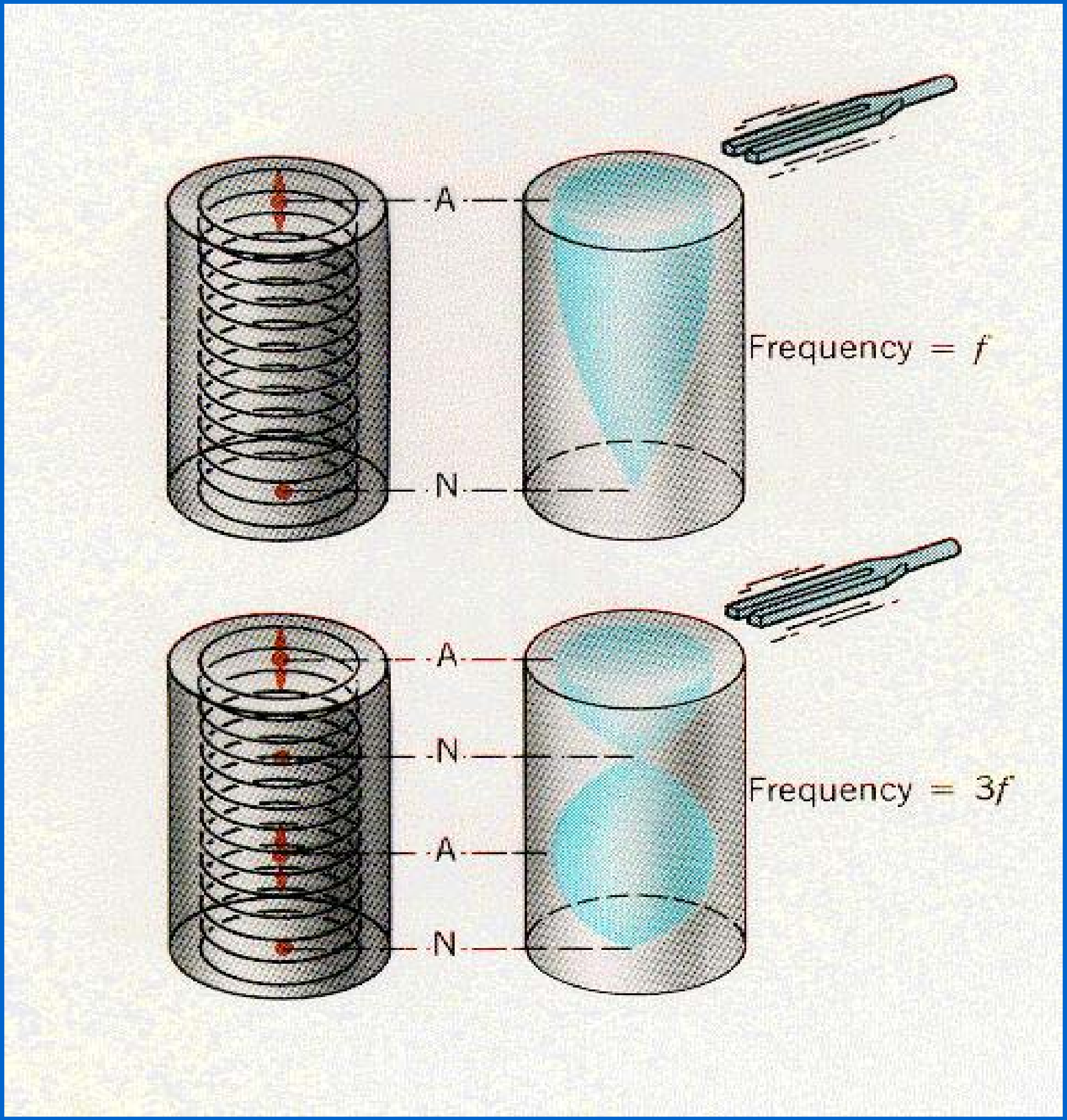
Which harmonics resonate?

- Ends are the same:
  - Multiples of f_1: f_1, $2f_1$, $3f_1$, $4f_1$…
- Ends are different:
  - Odd multiples of f_1: f_1, $3f_1$, $5f_1$, $7f_1$...



Both ends open:

anti-nodes at ends

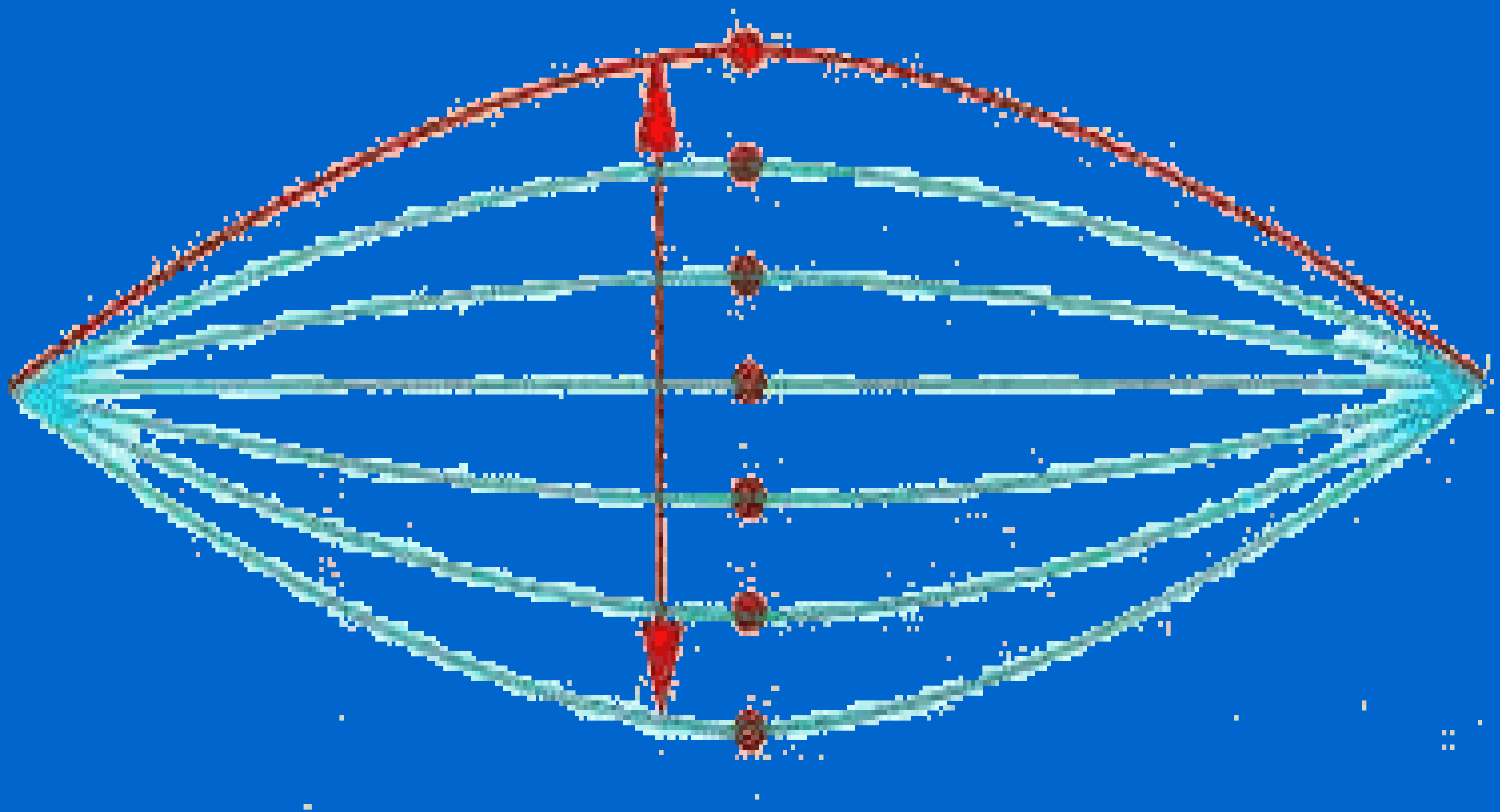


One end open--one end closed:

anti-node at one end--node at the other

Standing Waves:

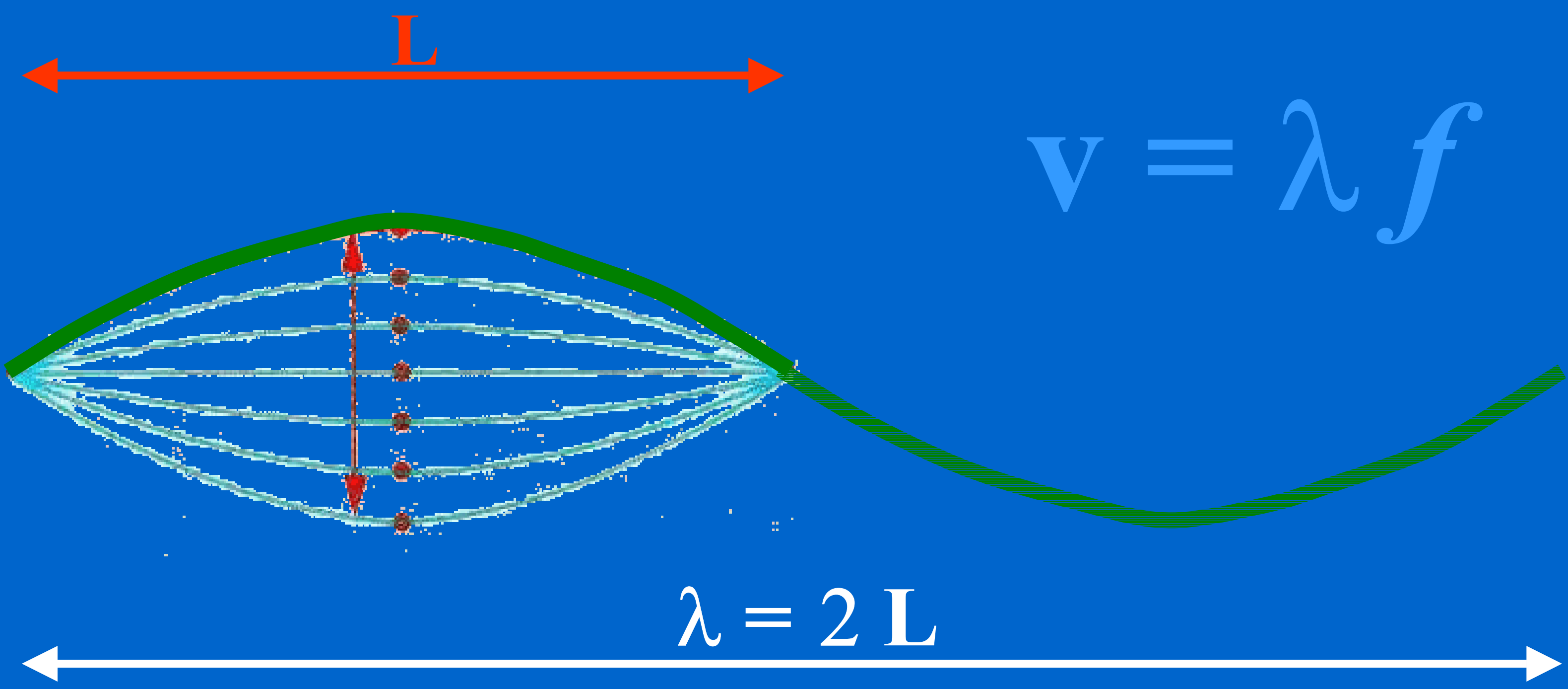
- Only happens at certain frequencies
  - Resonance phenomenon
  - Reflection and superposition of a wave
- <u>NODES:</u> points of zero vibration
- <u>ANTINODES:</u> points of maximum vibration



What is the wavelength?

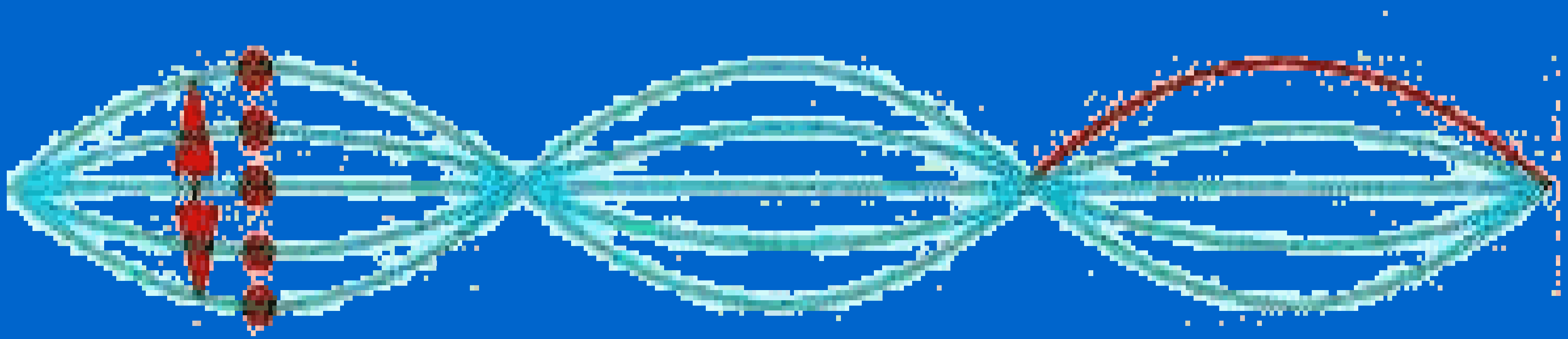
Rule: Each lobe is half a wavelength; # lobes x $\lambda/2 = L$

$$\mathbf{v} = \lambda f$$



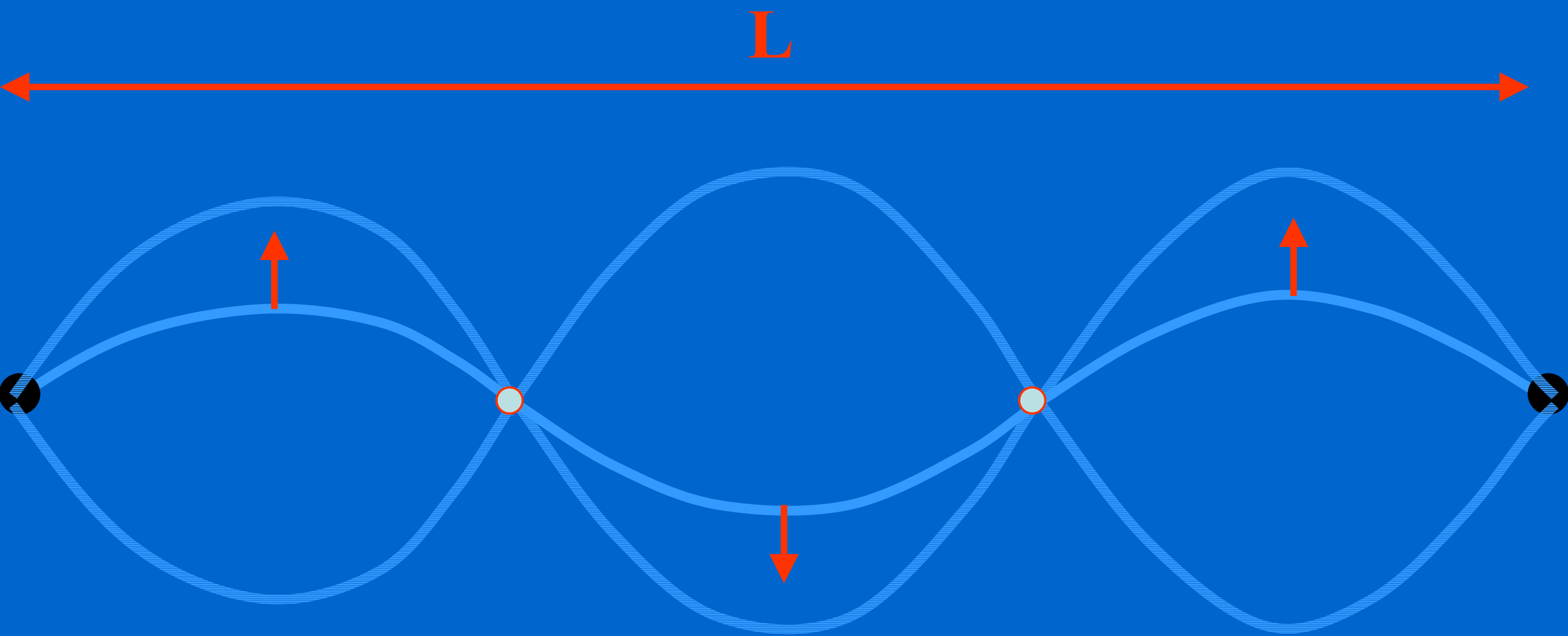
What is the frequency?

$$\mathbf{f} = \mathbf{v}/\ \lambda$$
$$= \mathbf{v}/\ 2\mathbf{L}$$



Snapshot

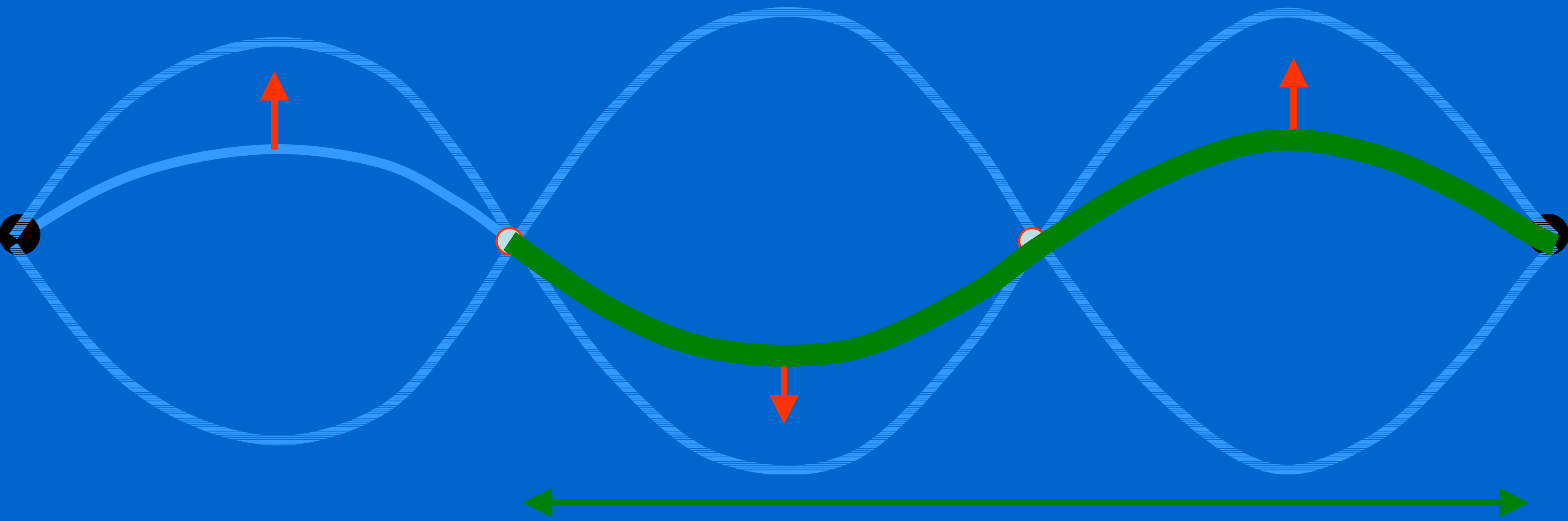
What is the wavelength?



L

$3 \quad \lambda/2 \quad = L$

Rule: Each lobe is half a wavelength; # lobes x $\lambda/2 = L$

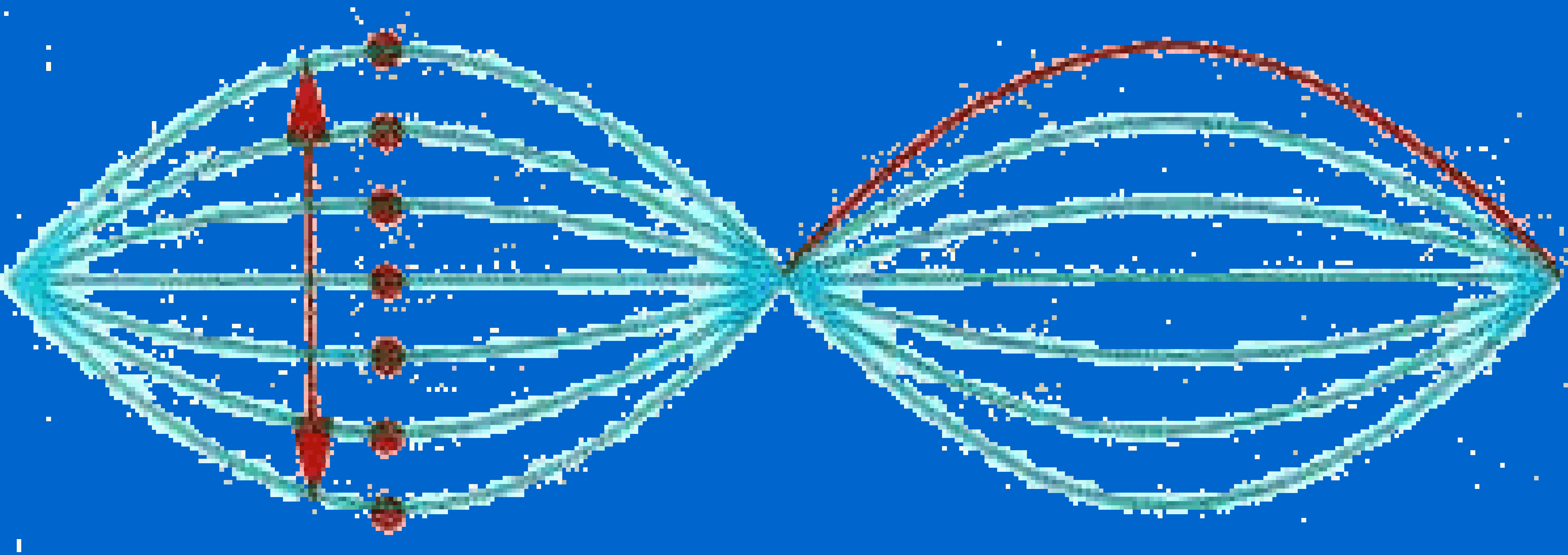


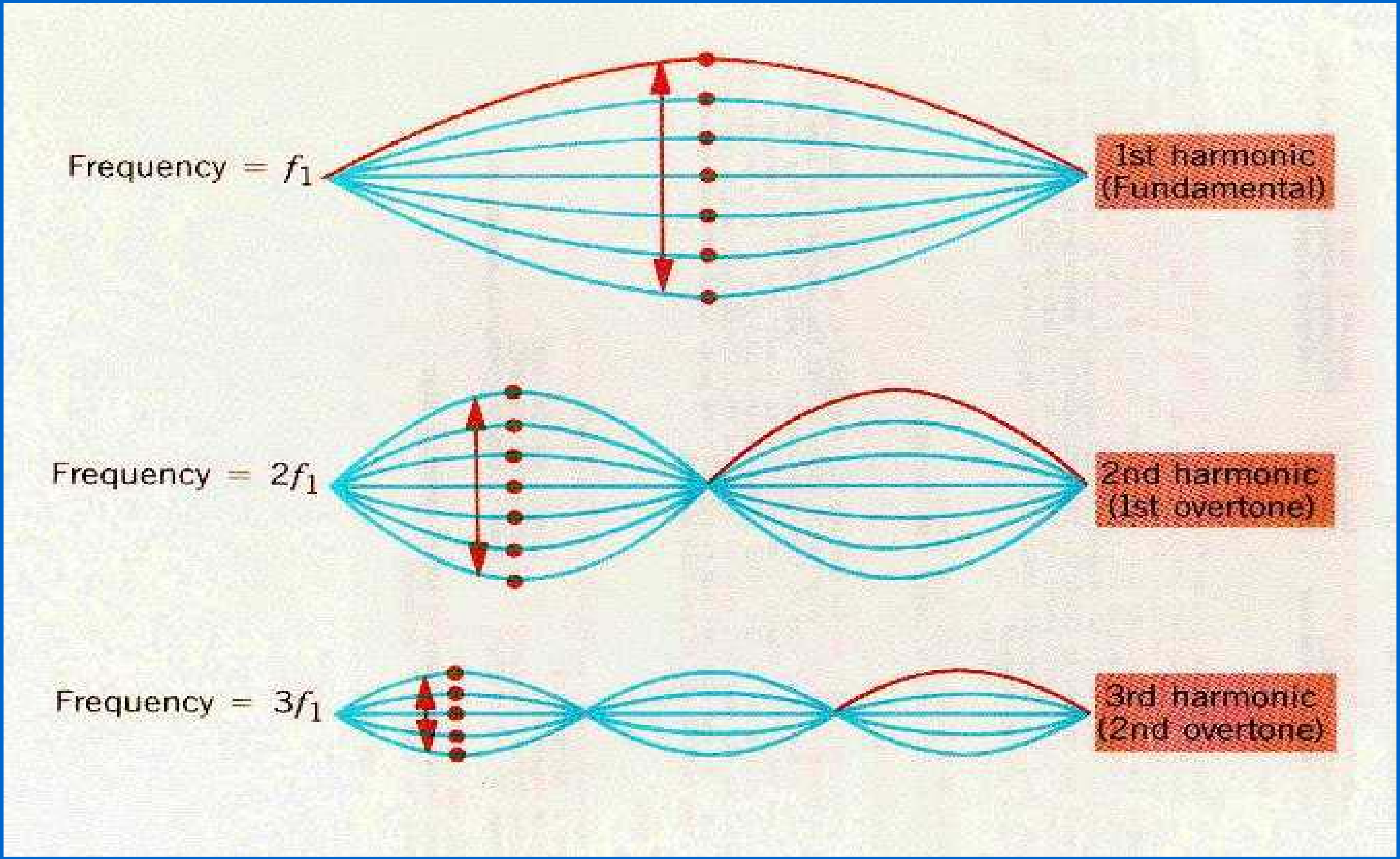
$\lambda = 2/3\ L$

$$\mathbf{f = v/\,\lambda}$$

$$= \frac{\mathbf{v}}{2/3\ L} \quad = \mathbf{3v/2L} \ = \mathbf{3(v/2L) = 3f_1}$$

What is the frequency of this mode?



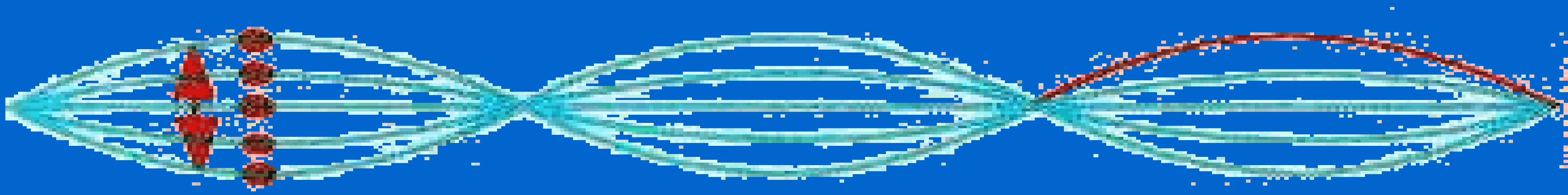

Frequency = f_1
1st harmonic
(Fundamental)
Frequency = $2f_1$
2nd harmonic
(1st overtone)
Frequency = $3f_1$
3rd harmonic
(2nd overtone)

A string stretched between 2 fixed supports sets up standing waves with 2 nodes between the ends when driven at 240 Hz.

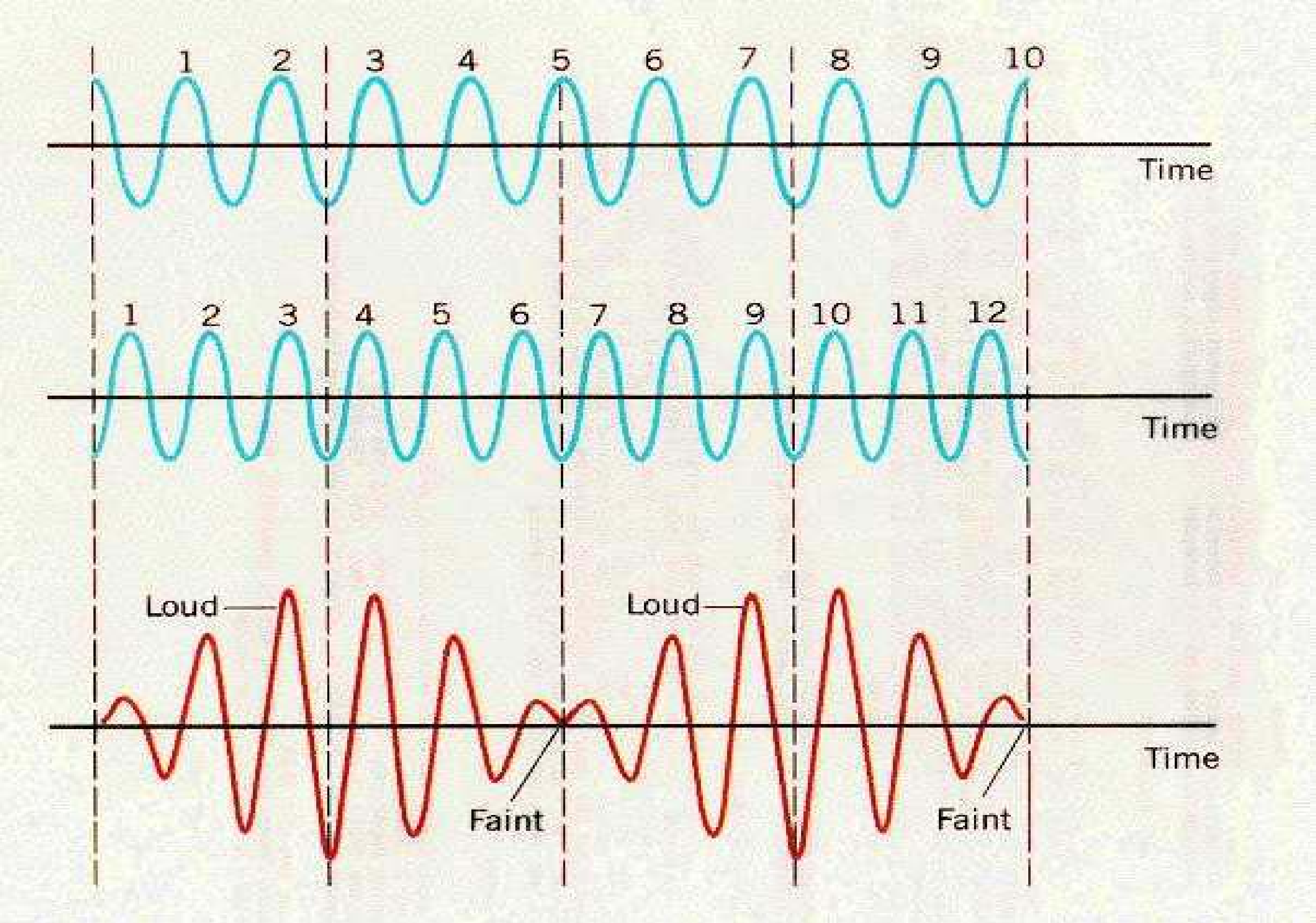
a. Draw a picture of the standing wave pattern.

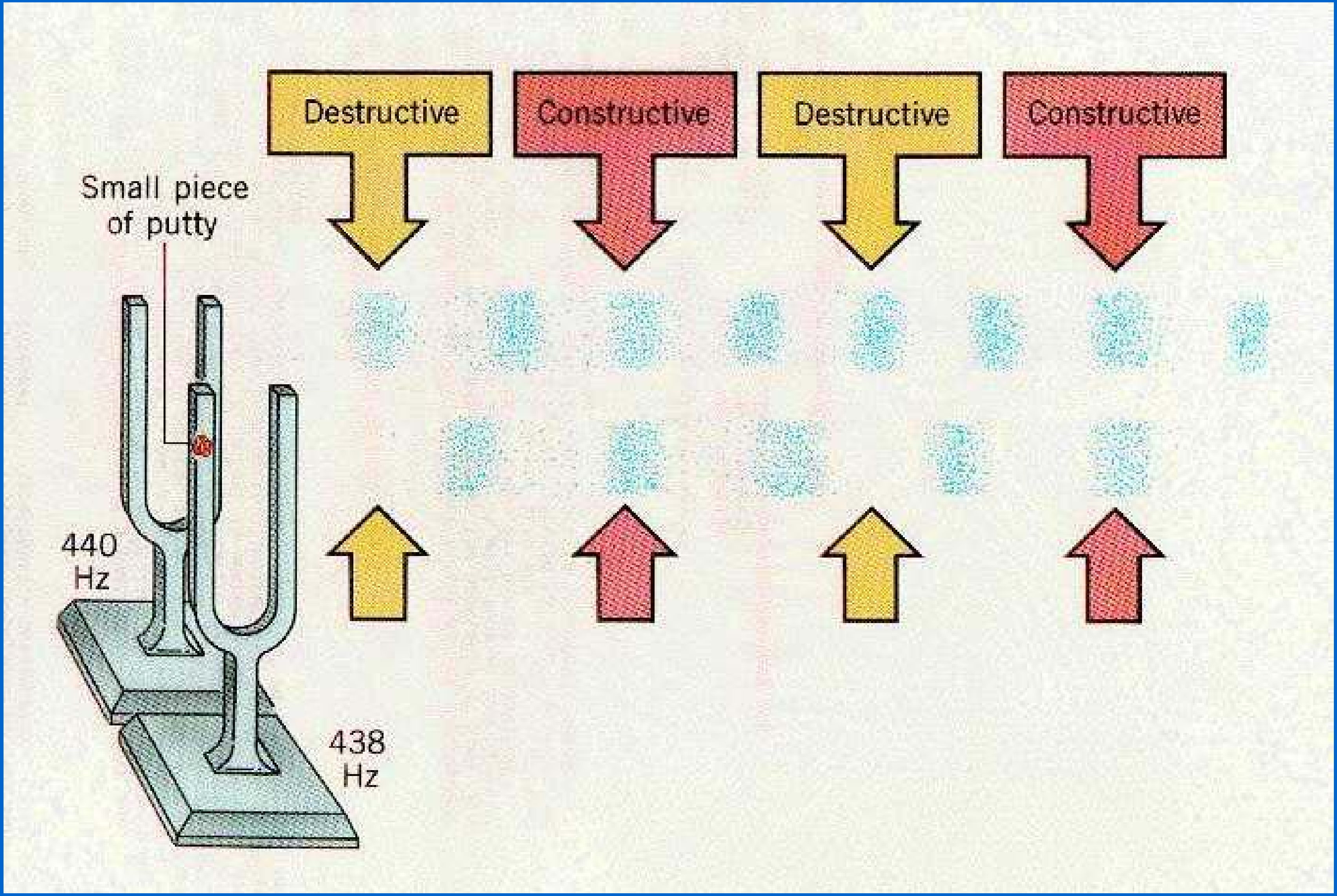
b. What is the fundamental frequency?

c. At what frequency will the standing wave have 3 nodes (between ends)?



BEATS

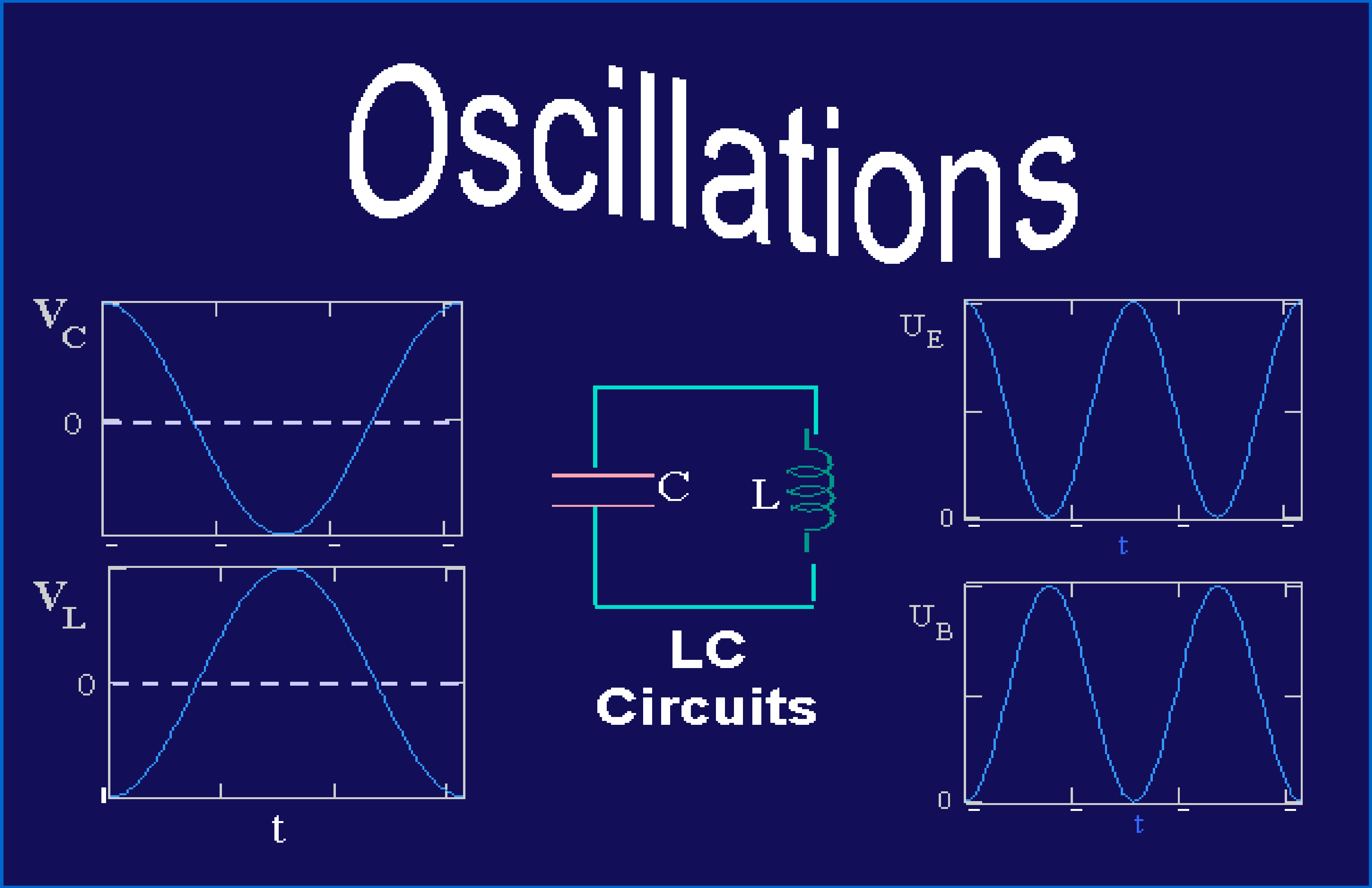

1
2
3
4
5
6
7
8
9
10
Time
1
2
3
4
5
6
7
8
9
10
11
12
Time
Loud
Loud
Faint
Faint
Time


Destructive
Constructive
Destructive
Constructive
Small piece of putty
440 Hz
438 Hz

BEATS:

- results from interference of two slightly different frequencies

$$f_{beat} = |f_2 - f_1|$$


Oscillations
V_C
0
V_L
0
t
C
L
LC
Circuits
U_E
0
t
U_B
0
t

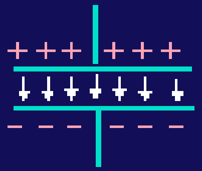
Lecture Outline

- Qualitative descriptions:
  - LC circuits (ideal inductor)
  - LC circuits (L with finite R)
- Quantitative descriptions:
  - LC circuits (ideal inductor)
    - Frequency of oscillations
    - Energy conservation?
  - LC circuits (L with finite R)
    - Frequency of oscillations
    - Damping factor

First, a bit of an energy review...

Energy in the *Electric* Field

Work needed to add charge to capacitor...



$$dW = dq(V) = dq\left(\frac{q}{C}\right)$$

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$ **... total work ...**

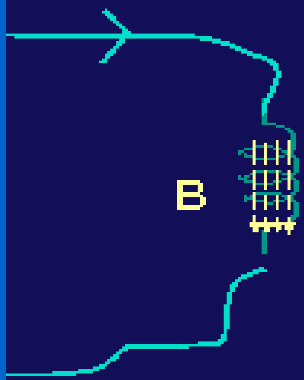
$$u = \frac{W}{\text{volume}} = \boxed{\frac{1}{2}\varepsilon_0 E^2}$$ **... energy density**

Energy in the *Magnetic* Field

"Power" accounting in a LR circuit...

$$\varepsilon I = I^2 R + LI\frac{dI}{dt}$$ Loop rule x I ...

$$P_L = \frac{dU}{dt} = LI\frac{dI}{dt}$$ Rate of energy flow into L



$$U = \int_0^U dU = \int_0^I LI\,dI$$ Total energy flow

$$\boxed{U = \frac{1}{2}LI^2}$$ ... energy stored

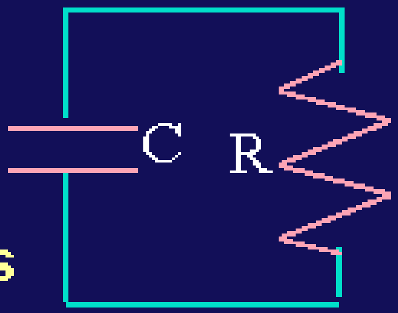
Energy Density:

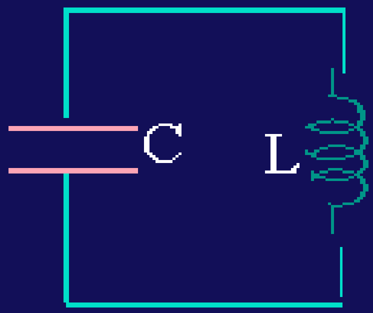
$$\boxed{u_{electric} = \frac{1}{2}\varepsilon_0 E^2}$$

$$\boxed{u_{magnetic} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

LC Circuits

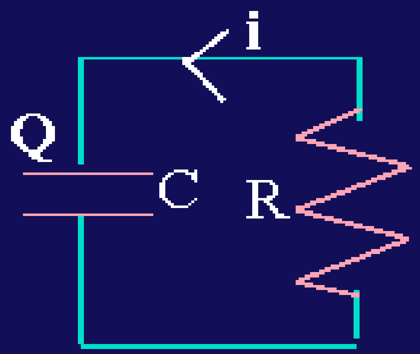
- Consider the LC and RC series circuits shown:

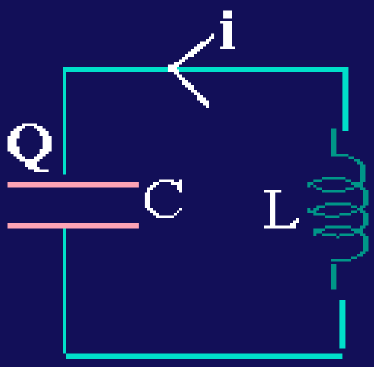




- Suppose that the circuits are formed at t=0 with the capacitor C charged to a value Q. Claim is that there is a qualitative difference in the time development of the currents produced in these two cases. Why??
- Consider from point of view of energy!
  - In the RC circuit, any current developed will cause energy to be dissipated in the resistor.
  - In the LC circuit, there is NO mechanism for energy dissipation; energy can be stored both in the capacitor and the inductor!

RC/LC Circuits



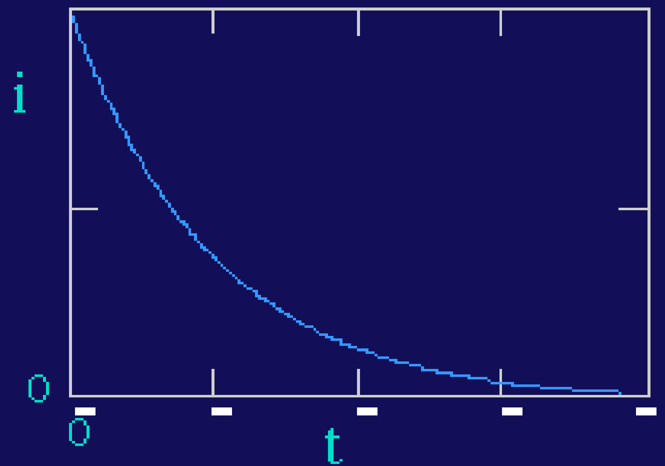


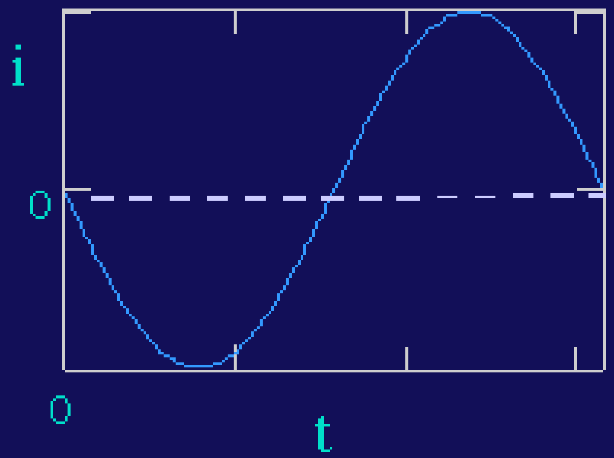
RC:

current decays exponentially

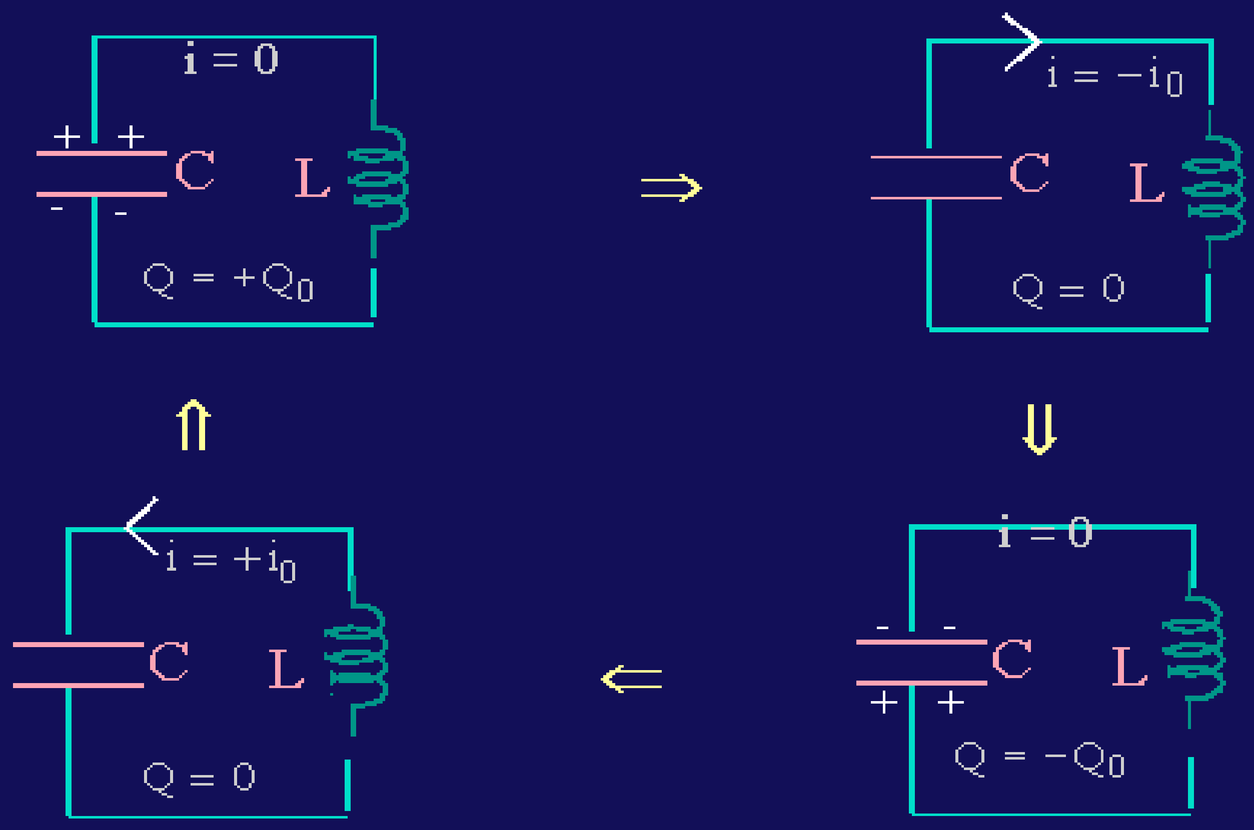
LC:

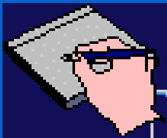
current oscillates



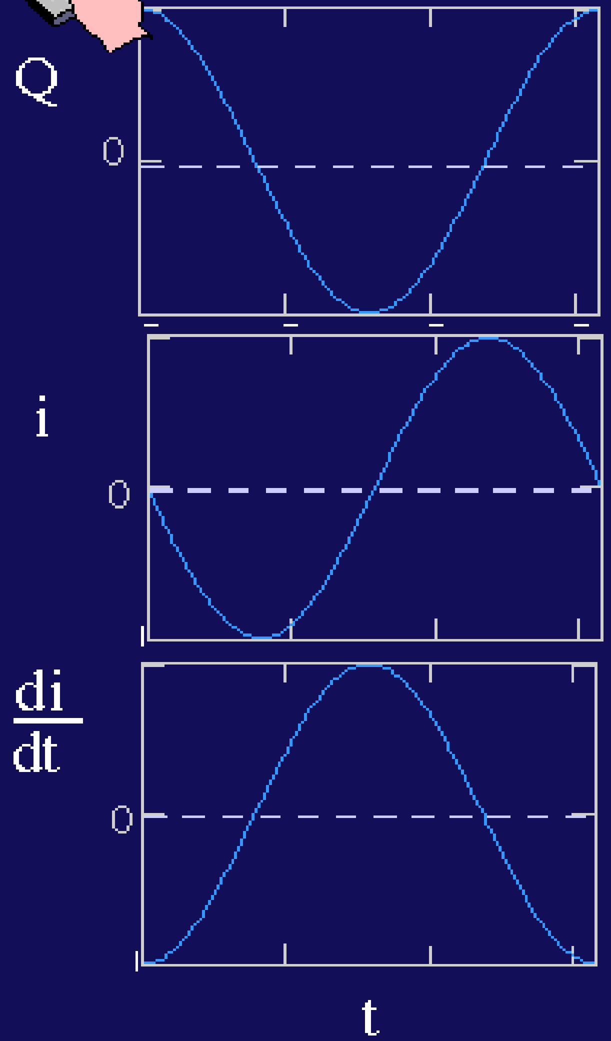


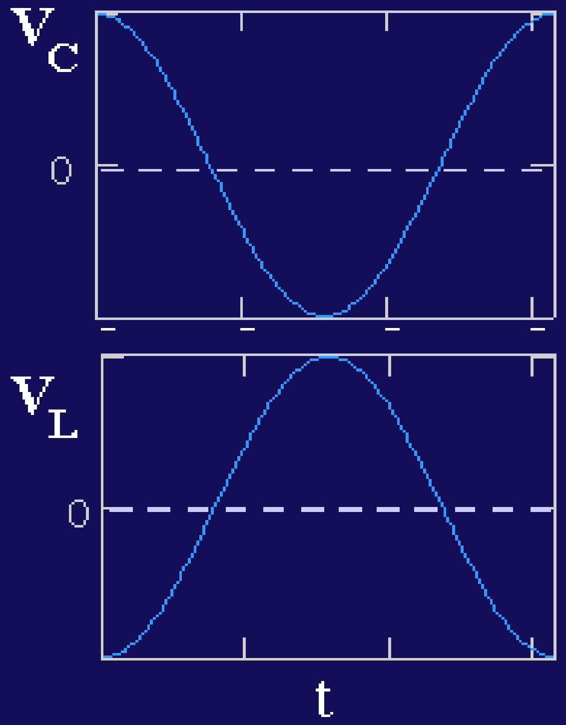
LC Oscillations (qualitative)





LC Oscillations (qualitative)

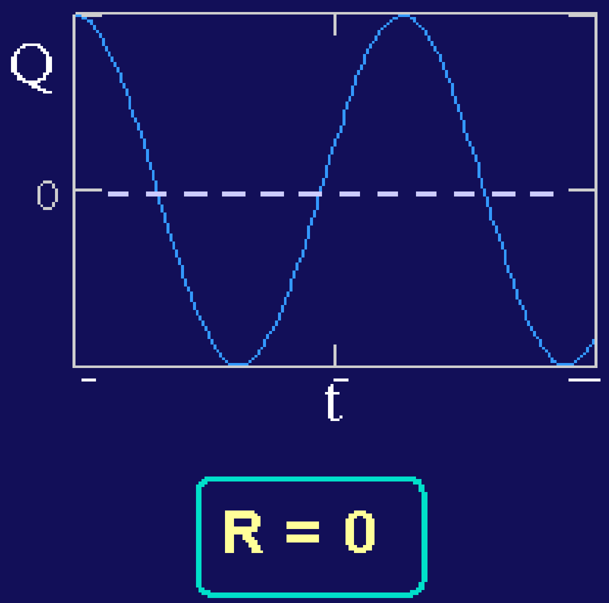


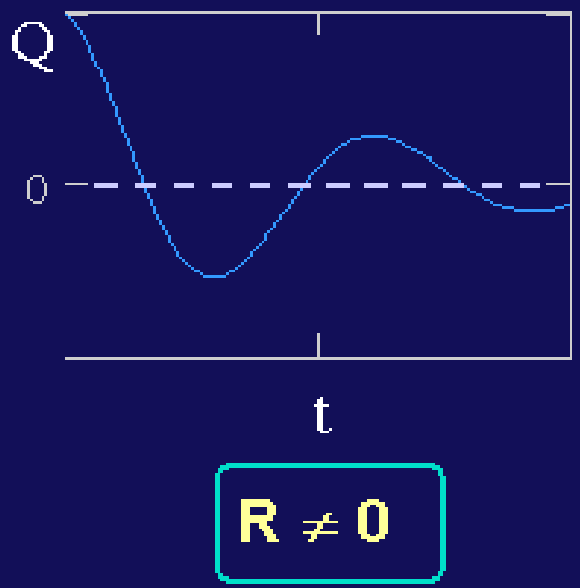


How do these change if L has a finite R?

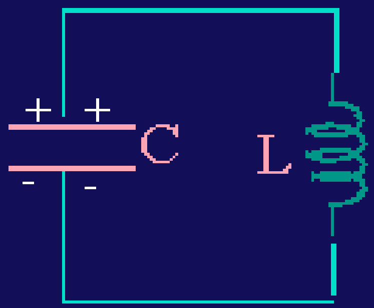
LC Oscillations (L with finite R)

- If L has finite R, energy will be dissipated in R and the oscillations will become damped.

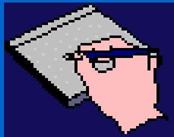




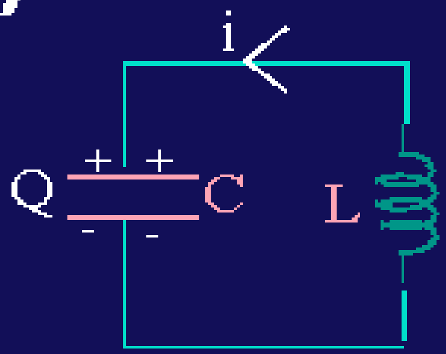
LC Oscillations (quantitative)



- What do we need to do to turn our qualitative knowledge into quantitative knowledge?
  - What is the frequency ω of the oscillations (when R=0)?
  - How does damping depend upon R?
  - Does R change the frequency?



LC Oscillations (quantitative)



- Begin with the loop rule:

$$\boxed{L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0}$$

- Guess solution: (just harmonic oscillator!)

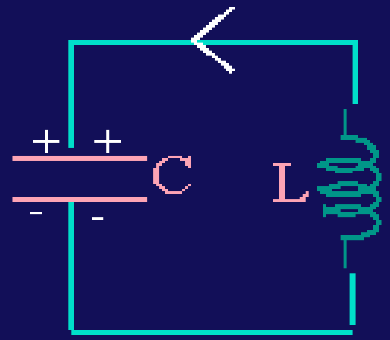
$$\boxed{Q = Q_0 \cos(\omega_0 t + \phi)}$$

remember: $\boxed{-kx = m\frac{d^2x}{dt^2}}$

where:
- ω_0 determined from equation
- ϕ, Q_0 determined from initial conditions

- Procedure: differentiate above form for Q and substitute into loop equation to find ω_0.

LC Oscillations (quantitative)



$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0$$

- General solution:

$$\boxed{Q = Q_0\cos(\omega_0 t+\phi)}$$

- Differentiate:

$$\frac{dQ}{dt} = -\omega_0 Q_0 \sin(\omega_0 t+\phi)$$

$$\frac{d^2Q}{dt^2} = -\omega_0^2 Q_0 \cos(\omega_0 t+\phi)$$

- Substitute into loop eqn:

$$L\left(-\omega_0^2 Q_0\cos(\omega_0 t+\phi)\right)+\frac{1}{C}\left(Q_0\cos(\omega_0 t+\phi)\right)=0 \quad \Rightarrow \quad -\omega_0^2 L+\frac{1}{C}=0$$

$$\therefore \quad \boxed{\omega_0=\frac{1}{\sqrt{LC}}}$$

which we could have determined from the mass on a spring result:

$$\omega_0=\sqrt{\frac{k}{m}}=\sqrt{\frac{1/C}{L}}=\frac{1}{\sqrt{LC}}$$

LC Oscillations Energy Check

- Oscillation frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ has been found from the loop eqn.
- The other unknowns (Q_0, ϕ) are found from the initial conditions. eg in our original example we took as given, initial values for the charge (Q_i) and current (0). For these values: $Q_0 = Q_i$, $\phi = 0$.
- Question: Does this solution conserve energy?

$$U_E(t) = \frac{1}{2}\frac{Q^2(t)}{C} = \frac{1}{2C}Q_0^2 \cos^2(\omega_0 t + \phi)$$

$$U_B(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}L\omega_0^2 Q_0^2 \sin^2(\omega_0 t + \phi)$$

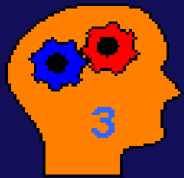
Energy Check

Energy in Capacitor

$$\boxed{U_E(t) = \frac{1}{2C} Q_0^2 \cos^2(\omega_0 t + \phi)}$$

Energy in Inductor

$$U_B(t) = \frac{1}{2} L \omega_0^2 Q_0^2 \sin^2(\omega_0 t + \phi)$$



$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \Downarrow$$

$$\boxed{U_B(t) = \frac{1}{2C} Q_0^2 \sin^2(\omega_0 t + \phi)}$$

Therefore, $\boxed{U_E(t) + U_B(t) = \frac{Q_0^2}{2C}}$